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# NUCLEAR INSTRUMENTS & METHODS

A JOURNAL ON ACCELERATORS, INSTRUMENTATION  
AND TECHNIQUES IN NUCLEAR PHYSICS

EDITOR: K. SIEGBAHN - UPPSALA

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*A journal on accelerators, instrumentation and techniques in nuclear physics*

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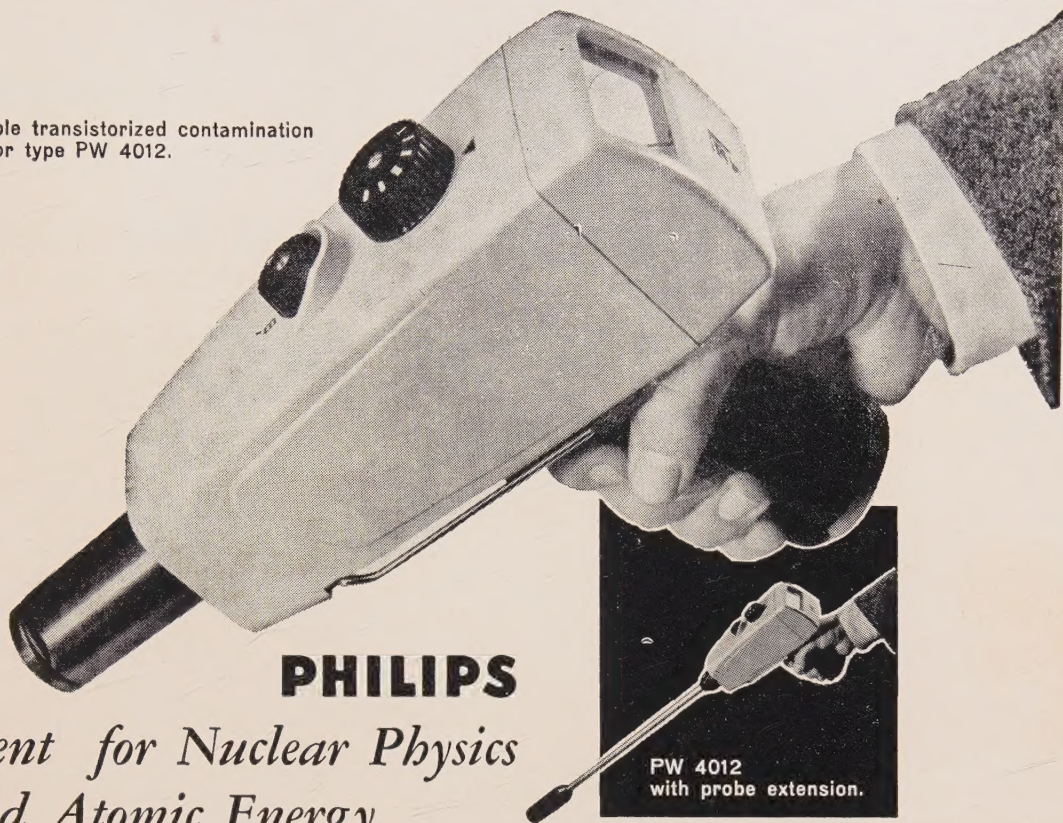
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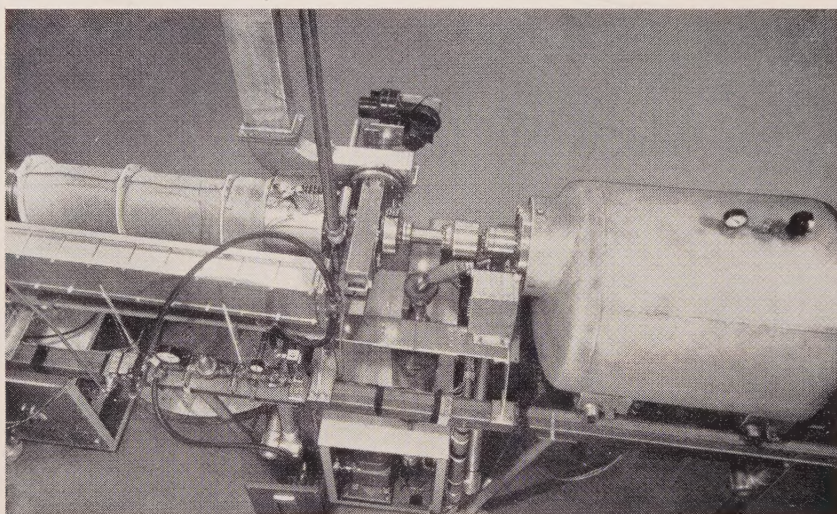
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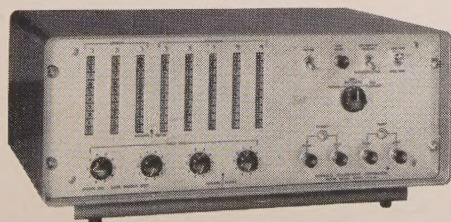
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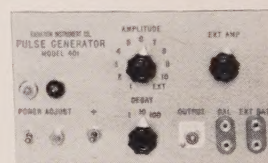
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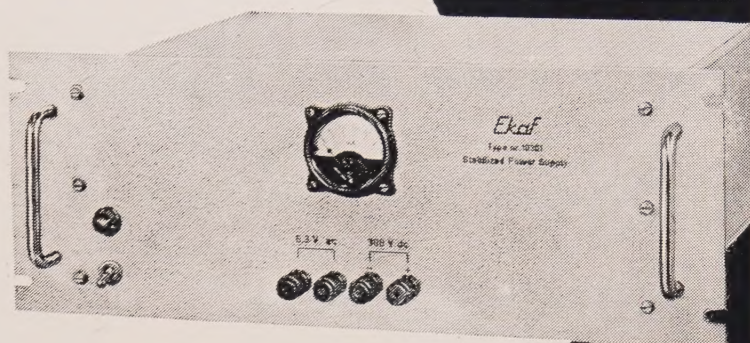
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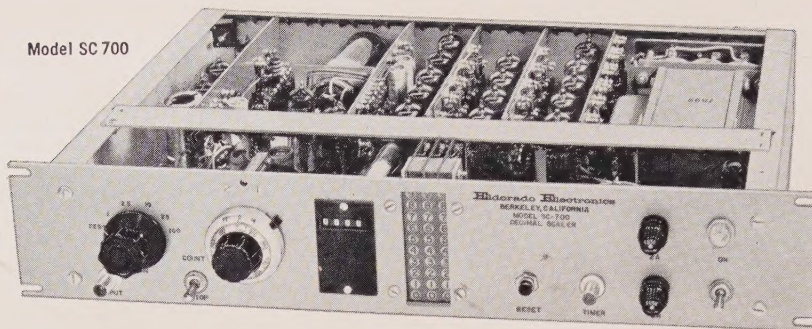
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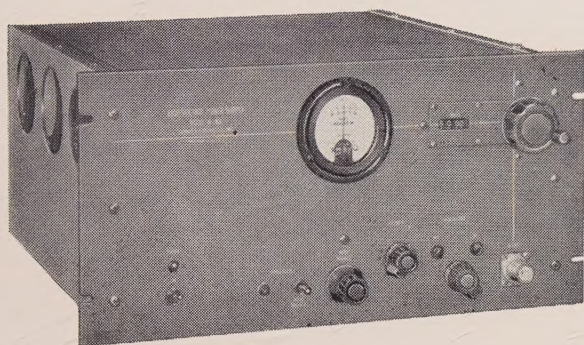
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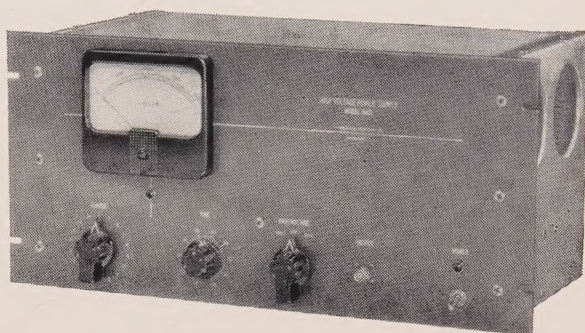
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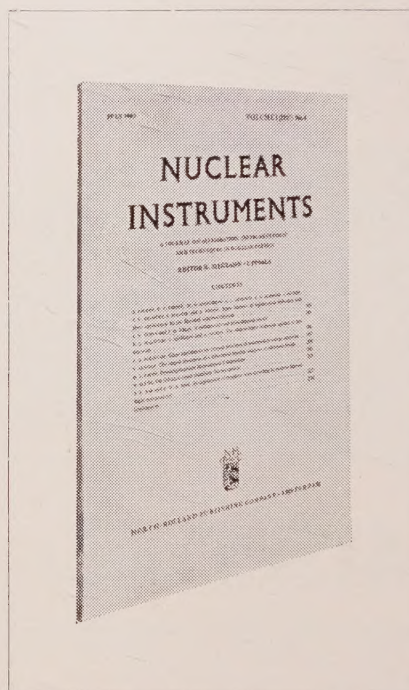
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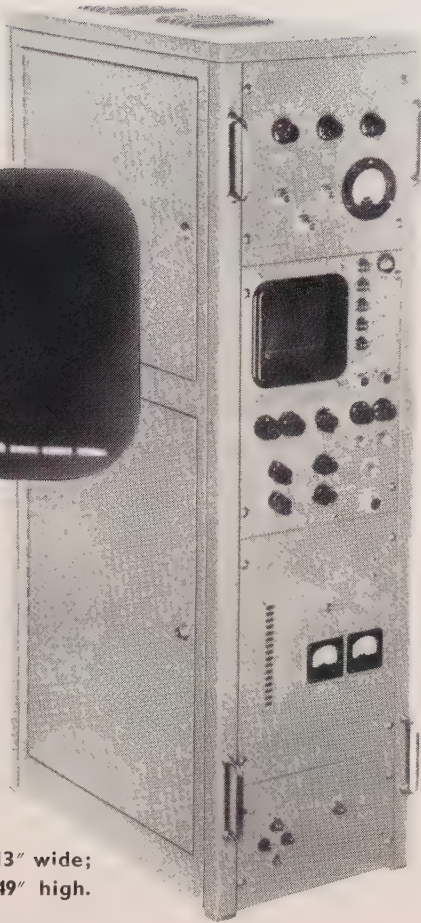
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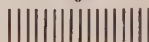


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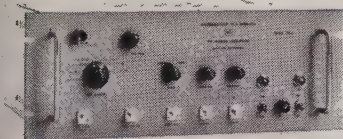
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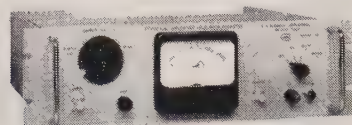
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# The place of the Particle Accelerator in Basic Research...

## The Effects of Ionizing Radiation in Liquid Systems - V

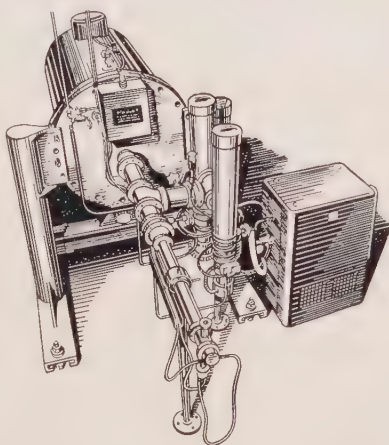
Determination of the effects of ionizing radiation in liquid systems is a challenging problem. Many physical models of the liquid phase have been proposed, and some success has attended their use. Although short-range molecular order appears to exist, complete disorder occurs at distances substantially greater than the molecule size, where the individual molecules wander through the system in a random manner<sup>1</sup>. This condition, combined with the difficulty of measuring directly the changes in physical and chemical properties of the liquid molecules, complicates the analysis of ionizing radiation effects in liquid systems.

### Experimental Techniques

The experimental techniques usually adopted to study radiation effects are: measurement of the change of some solute in solution, or measurement of the yield of some product formed by the action of radiation. Analyses of radiation-formed products can be used to develop a mechanism for explaining the interaction of the incident radiation. Several Van de Graaff® accelerators, including those at the University of Notre Dame and the Brookhaven National Laboratory, are presently being used in studies of liquids under electron or x-ray bombardment.

### Water Studies

One of the best-known liquids, which has received a great deal of attention from radiation chemists because of its importance in many radiobiological and radio-



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chemical systems, is water. The generally accepted result of the interaction of a charged particle with a water molecule is dissociation, resulting in the production of free H and OH radicals. These may combine to re-form water or the molecular products  $H_2O_2$  and  $H_2$ , or they may diffuse from the site of their formation to react with solute molecules in solution<sup>2</sup>. Studies of the kinetic behavior of these radicals have been important in analyzing many radiation-induced reactions.

A typical experimental method of studying the effects of these radicals is to investigate the radiation yield in the wet and dry states. This will give the percentage of radiation-induced changes caused by the transfer of radiation energy from the solvent molecules to the solute. The addition of scavenger molecules which compete with the solute

for radicals will give some indication of the relative reactivity of the radicals.<sup>3</sup>

### Energy Levels

Since the energy of dissociation of typical liquid molecules is of the order of tens of electron volts, while the energy transferred from the incident radiation may be many hundreds of electron volts, it is difficult to measure the direct effect on the individual molecules. Although values of ionization and excitation energy have been extrapolated from gas-phase studies, it is not easy to relate the effect of the molecular binding energy in the solution to the dissociation energy. About the best that can be done is to obtain as uniform as possible a distribution of radiation energy throughout the system and to assume that all secondary electrons have the same probability of interaction as the primary beam. The yield is then calculated in terms of the number of molecules changed per unit-energy input. The unit is the so-called G value which measures the number of molecules changed or formed per 100 electron volts.

Although the analysis of radiation-induced changes in liquid systems is complex, great strides have been made during the past few years. *With the availability of homogeneous beams of charged particles from Van de Graaff accelerators, which permit wide selection of energy and current values, it is expected that even greater success will be attained in the near future.*

### References:

1. R. H. Fowler, N. B. Slater, *Trans. Far. Soc.*, **34**, 81, 1937
2. A. O. Allen, *Rad. Res.*, **1**, 85, 1954
3. H. Schwartz, *J.A.C.S.*, **77**, 4960, 1955

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## A VAN DE GRAAFF ACCELERATOR TUBE OF VERY LOW RETROGRADE ELECTRON CURRENT

D. R. CHICK, S. E. HUNT, W. M. JONES and D. P. R. PETRIE

*Associated Electrical Industries Limited, Aldermaston Court, Aldermaston, Berks.*

Received 3 August 1959

The upper limit to the voltage attained by Van de Graaff accelerators is often set by the flow of electrons towards the positive terminal of the tube, producing large X-ray fluxes and ionisation currents in the gas between the high voltage terminal and the earthed pressure vessel.

### 1. Introduction

In general the upper limit to the performance of electrostatic generators is determined by the behaviour of the accelerator tube. The machine at this Laboratory, operated as a voltage generator without a tube, has been shown to be capable of reaching a top terminal potential of 5.5 MV, but the maximum voltage obtained with an accelerating tube has been about 3.8 MV<sup>1)</sup>.

The influence of the accelerator tube on machine performance is complex. Under certain conditions the upper limit is set by retrograde electrons in the tube, which produce X-rays on striking its positive terminal.

In June 1953 the authors demonstrated that in the presence of such X-rays, relatively high leakage currents flow in the insulating gas from the terminal to the earthed pressure vessel. The leakage current was found to be proportional to gas pressure and to the electron current in the tube. The effect is due to ionization of the gas by the X-rays, the ions then being swept by the electric field to the earthed vessel walls. Independent work of a similar nature was reported by C. M. Turner<sup>2)</sup>.

In the presence of these leakage currents, the total current required at the machine terminal may then be such that the charge carrying capacity of the belt is exceeded and discharges appear along the length of the belt, or at the charging spray needles. This limiting condition may be reached at voltages far below

those necessary to produce major spark discharges in the tube, but they nevertheless represent a limitation introduced essentially by the behaviour of the accelerator tube.

By designing the internal electrodes of the tube to trap retrograde electrons a voltage of 3.7 MV has been obtained across a nine foot long tube, with immeasurably small electron currents. The performance of this tube is compared with that of tubes employing other internal electrode designs.

By using materials of low atomic weight at the top of the tube, the X-ray intensity produced by a given retrograde electron current should be reduced. C. M. Turner<sup>2)</sup>, using a composite beryllium lead anode assembly, reported on improved tube performance. Independent work by the authors in June 1954 demonstrated the decrease of ionization current resulting from the use of 1 in. thick beryllium anode.

Reversed electrostatic fields have been employed over sections of the tube in order to break the secondary electron multiplication chain<sup>3)</sup> and it has also been suggested that magnetic fields could be used to divert the electrons in order to prevent cascade multiplication at the electrode surfaces.

The application of suppressive electrostatic fields at the bottom and top of the tube have also been found to reduce retrograde currents<sup>4)</sup>.

A beryllium plate was fitted to the top of the tubes described in the present work, but the main effort has been concentrated on arresting the return electron multiplication chain by the geometrical design of the tube electrodes.

<sup>1)</sup> D. R. Chick and D. P. R. Petrie, *Proc. I.E.E.*, B 103, No. 8 (1956) 132.

<sup>2)</sup> C. M. Turner, *Phys. Rev.* 95 (1954) 599.

<sup>3)</sup> K. Firth and D. R. Chick, *J. Sci. Instr.* 30 (1953) 117.

<sup>4)</sup> E. K. Inall, *Proc. Phys. Soc. B* 63 (1950) 1068.

The A.E.I. Van de Graaff accelerator is particularly well suited for investigations of tube breakdown, in that current arriving at the top plate of the tube, current leaving the high voltage terminal and all other currents flowing into or out of the high voltage terminal can be measured using the telemetering circuits described by Millar<sup>5</sup>). Ionisation current flowing to the pressure vessel surrounding the machine is measured by insulating this vessel and earthing it through a protected meter, and the intensity of the X-radiation is measured by means of a Geiger Muller tube with lead shield, mounted in one of the pressure vessel windows immediately opposite the top of the accelerator tube.

## 2. The Mechanism of Tube Breakdown

Several investigations into the behaviour of evacuated accelerator tubes under conditions of high electrical stress have been reported<sup>6,7,8</sup>). Although it is not claimed that the detailed mechanism of tube breakdown is completely understood, the results obtained at this Laboratory indicate that conduction in tubes can best be explained in terms of two separate mechanisms.

In one case partial breakdown occurs for a short time, the machine is partially discharged through the tube so that the top terminal potential falls slightly and the pressure in the tube rises momentarily. Current pulses can be observed on a collector placed at the base of the tube. This type of breakdown has been explained by Harris<sup>9</sup>) in terms of a self-sustaining secondary emission process between the two ends of the tube. Positive particles strike the bottom of the tube and produce neutrals, which strike the top of the tube and produce positive ions. This continues until the tube potential falls to a value at which the process is no longer self-sustaining. Mansfield and Fortescue<sup>10</sup>) conclude that both negative and positive ions take part in the process. The second main mechanism of tube conduction appears to be secondary electron multiplication at the surface of the electrodes. Appreciable negative currents are observed at the top of the accelerator tube, but

the voltage obtainable across the tube remains steady. It is possible that negative ions as well as electrons play a large part in this secondary emission process.

In designing tube electrodes, therefore, efforts must be made to reduce conduction due to either of the above mechanisms. The probability of end-to-end breakdown can be reduced by limiting the solid angle which the top of the tube subtends at the bottom.

This is achieved in the "double waisted" design described in section 7.2. The concept of "waisting" is already widely used on the design of accelerator tubes.

In addition to this the "inverted cone" design described in section 7.3 provides a series of geometrical traps which prevent secondary electrons produced on the electrode surfaces from proceeding up the tube, and in comparison with the simple "double waisted" design this has been shown to be very effective in reducing the steady retrograde tube current.

## 3. Construction of the Accelerator Tube

The construction of the shell of the accelerator tube has been described previously<sup>1</sup>), except that for the second two electrode systems mentioned in section 4, the tube was cemented using Vinyl Seal instead of Araldite. Briefly the tube was made up of porcelain rings of internal diameter 10 ins. and external diameter 12 ins. These are corrugated and glazed on the outside and have cylindrical sandblasted surfaces inside. The rings were sealed to flat stainless steel diaphragms, using Vinyl Seal cement applied in such a way that surplus material formed a bead on the outside of the joint but not on the inside. The tube was jig assembled to ensure that it was straight, the jigs being located on the outer edges of the diaphragms. The inner

<sup>5</sup>) B. Millar, K. Firth and D. R. Chick, *J. Sci. Instr.* **32** (1955) 120.

<sup>6</sup>) L. Cranberg and J. B. Henshall, *J. App. Phys.* **30** (1958) 708.

<sup>7</sup>) R. L. Fortescue, *Progr. in Nuclear Physics* **1** (1950) 21.

<sup>8</sup>) L. Cranberg, *J. Appl. Phys.* **23** (1952) 518.

<sup>9</sup>) D. J. Harris, *Queen Mary College Ph. D. Thesis* (1953).

<sup>10</sup>) W. K. Mansfield and R. L. Fortescue, *Brit. J. App. Phys.* **8** (1957) 73.

edge of the diaphragms supported the internal tube electrodes, which were removable. The diaphragms were spaced  $2\frac{9}{16}$  in. apart, center-to-centre, and the total stressed length of the tube was nine feet.

#### 4. Internal Electrodes

Three internal electrode systems were tested. The first consisted of toroidal stainless steel rings of outside diameter 9 in. and inside diameter 7 in. as shown in fig. 1. The toroidal electrodes were originally adopted as an improvement on the earlier dished electrodes<sup>1)</sup>, in that they produce a more uniform field within the porcelain. In the second series of tests these

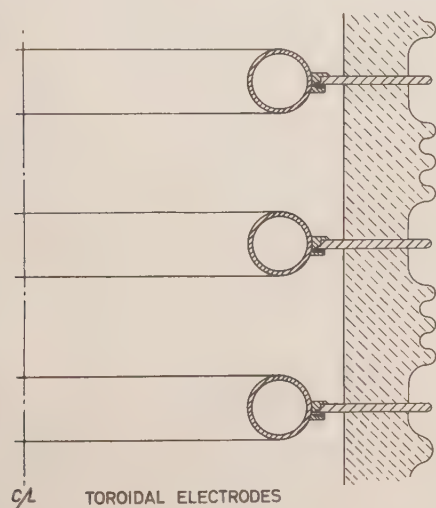


Fig. 1

were used to support internal stainless steel diaphragms, which had apertures of diameter varying from 6 in. to 2 in. and were arranged to form a "double waisted" configuration as shown in fig. 2. In order to maintain adequate pumping speed at the top of the accelerator tube, all diaphragms with apertures of diameter three inches or less also had four  $1\frac{1}{2}$  in. diameter pumping holes, which were staggered in alignment so that they did not provide a direct path for retrograde negative particles.

The third internal electrode configuration consisted of the same toroids and internal spinnings, re-arranged so that the inner edges of the apertures formed a series of truncated cones,

with the apertures of smallest diameter nearest the negative potential end of the tube (also shown in fig. 2). The discontinuities at the junction between each conical section were designed to arrest the flow of secondary electrons up the tube. The top conical section had the

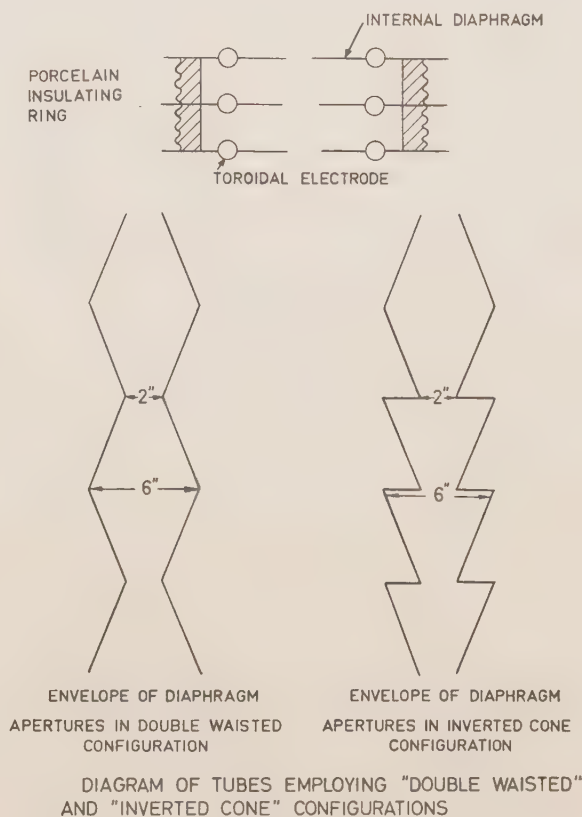


Fig. 2

small diameter apertures near the top of the tube to provide better shielding of the beam from stray electrostatic charges on the walls of the accelerator tube (section 8.1). In later tests it was found that shielding was further improved by replacing the upper four electrodes by dished electrodes of the type described by Chick and Petrie<sup>1)</sup>.

#### 5. Vacuum System

The accelerator tubes were continuously pumped by two 500 l/sec oil diffusion pumps which were connected to the base of the tube by a large bore steel tube, and were backed by a rotary pump. To avoid contamination of the

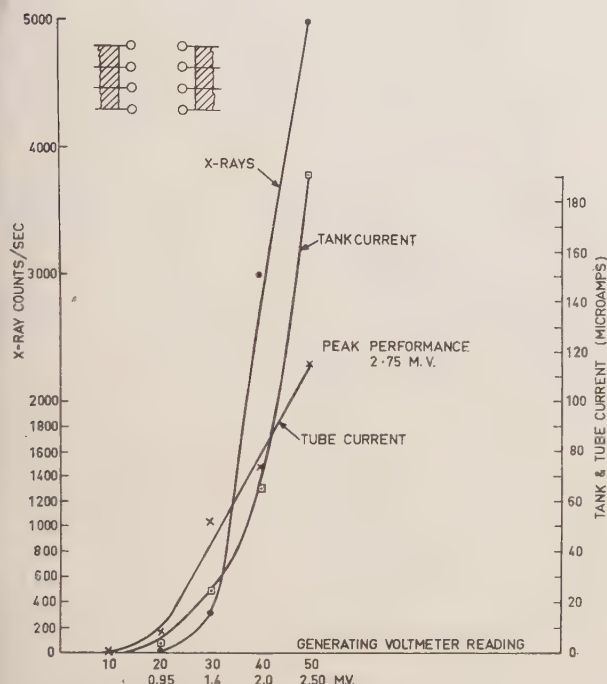


## 7.2. TUBE USING FLAT INTERNAL DIAPHRAGMS IN A "DOUBLE WAISTED" CONFIGURATION

The tube current, tank current, and X-ray intensity recorded using this tube are plotted in fig. 5. These became measurable at about 2.0 MV. The maximum steady voltage obtainable was approximately 3.3 MV, at which

5  $\mu$ A. This may well have been corona rather than ionization current.

The upper limit to the voltage obtainable with this configuration was set by the dielectric strength of the porcelain rings, three of which were punctured through their volume after the generator had been running at about 3.7 MV for some hours.



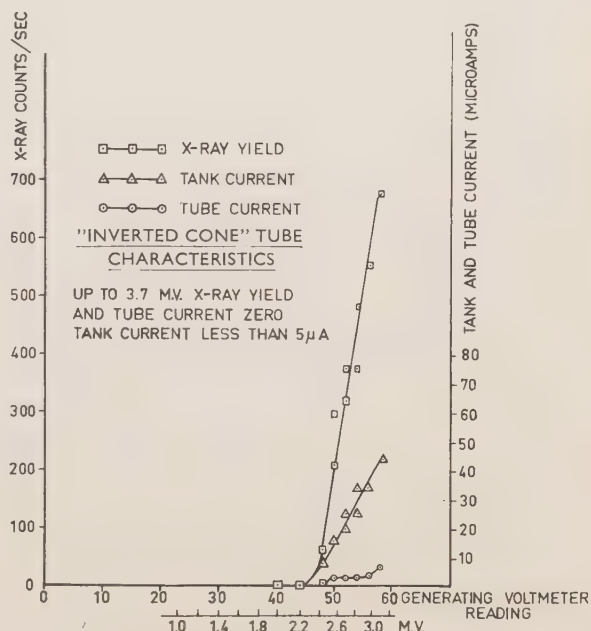
TOROIDAL ELECTRODE TUBE CHARACTERISTICS

Fig. 4

voltage the tank and retrograde electron current loading were such as to produce occasional breakdown along the length of the charging belt. Pulsed type partial breakdowns also occurred at this voltage, and two porcelain rings were also punctured through their volume. The positional stability of the beam was much improved.

## 7.3. TUBE USING FLAT INTERNAL DIAPHRAGMS WITH "INVERTED CONE" CONFIGURATION

The performance of this tube differed from all others in that retrograde electron current and top terminal X-rays were immeasurably small up to 3.7 MV, and the current flowing to the pressure tank at this voltage was only



DOUBLE WAISTED TUBE CHARACTERISTICS

Fig. 5

It was found that by replacing the top four electrodes by stainless steel dished electrodes, the beam focussing properties were improved. A beam spot of a few millimetres in diameter which was positionally stable to a few millimetres, was obtained.

## 8. Discussion of Results

### 8.1. POSITIONAL STABILITY OF THE BEAM

The large fluctuations in beam position observed using the toroidal electrons alone were attributed initially either to the interaction of the large retrograde electron currents with the proton beam, or to the stray charges on the walls of the porcelain tube deflecting the proton beam. It was observed that the positional

stability did not correlate very closely with the recorded retrograde current.

Plots of the equipotential lines using "Teledeltos" conducting paper as described by Milner<sup>11)</sup> for systems simulating the simple toroidal electrodes and toroidal electrodes with internal diaphragms have shown that the latter greatly improve the shielding of the beam from wall charges. It was for this reason that diaphragms with small holes were used at the top of the tube, where the less energetic beam was more susceptible to deflection by electrostatic fields. Even better shielding was provided by the dished type of electrodes and for this reason the top four electrodes were replaced by this type in later tests.

## 8.2. RETROGRADE ELECTRON SUPPRESSION

The high retrograde negative current observed with the simple toroidal electrodes can be attributed to a secondary multiplication process occurring at the electrode surfaces. The very high magnitude of this current is thought to be due to the fact that the electrodes present large areas where the surface is nearly parallel to the direction of flow of negative particles. The performance of this tube was characterised by the lack of pulsed voltage breakdown, probably because the electron multiplication process at the electrode surfaces reduced the voltage attainable to a level where this type of breakdown did not occur.

When plane diaphragms were installed in the "double waisted" configuration, the area of electrode surface at which progressive secondary multiplication could take place was considerably reduced, and the retrograde electron currents were correspondingly smaller. Pulsed breakdown occurred with this tube since the voltage gradients obtainable were higher.

The geometry of the "inverted cone" tube was such that the chain of the surface electron multiplication process was broken at the sudden discontinuity in the electrode apertures and consequently retrograde negative currents were negligible. The main limiting factor was the

volume puncture of the porcelain rings, but there was also evidence of slight pulsed breakdown at the higher voltages. This may be identified with the end-to-end breakdown mechanism, but subsequent examination of the tube showed that some inter-electrode sparking had also taken place. The voltage at which the onset of the pulsed partial breakdowns occurred was increased as the tube was run at high voltage gradients and the ultimate limit of the tube performance may have been higher, had insulators of higher dielectric strength been available.

It therefore appears that by suitably varying the apertures in the electrodes one of the main limitations on tube performance, that of electron multiplication at the electrode surfaces, has been eliminated. This is illustrated by comparing the performance of the "double waisted" tube and that of the "inverted cone" configuration tube (see fig. 5). These are geometrically identical so far as any end-to-end type of breakdown mechanism is concerned, but the latter is much superior in terms of trapping return electrons due to electrode surface multiplication.

This tube was also shown to be superior to the more conventional type of dished electrode tube, typical examples of which have been reported previously<sup>1)</sup>. Tests on this type of tube showed that retrograde electron current, top terminal X-rays and ionization current curves were similar to those of the double waisted tube shown in fig. 5.

## Acknowledgements

The authors would like to acknowledge the work of Dr. A. C. L. Barnard, Messrs. F. Cheetham, W. W. Evans, D. A. Hancock and G. Hunt, who collaborated in the building and testing of tubes, and members of the Design Office of this Laboratory, who contributed substantially to the detailed design.

Thanks are also due to Dr. T. E. Allibone, F.R.S., Director of the Research Laboratory, Associated Electrical Industries, Limited, for his interest, encouragement and permission to publish this paper.

<sup>11)</sup>C. J. Milner, B.T.H. Research Report No. L. 4021 (1951).

# STUDY OF TWO-DIMENSIONAL NON-LINEAR OSCILLATIONS BY MEANS OF AN ELECTROMECHANICAL ANALOGUE MODEL, APPLIED TO PARTICLE MOTION IN CIRCULAR ACCELERATORS

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Received 25 June 1959

The apparatus consists of a flexible quartz pendulum giving a perfect harmonic oscillation. The pendulum oscillates in a vacuum and hardly any damping takes place. Its surface is rendered conducting by metallization so that electrical forces can be applied to it. If required a permanent non linear characteristic can be given to the pendulum by this means. It can be excited into resonance by sinusoidal

forces, which may or may not depend on its instantaneous position. The paper describes the properties of the apparatus and experiments made by exciting the pendulum with perturbations at a fixed frequency. The performance of the machine was found to be such that one can regard it as a small calculating machine (analogue computer).

## 1. Introduction<sup>†</sup>

For the study of non-linear effects on betatron oscillations in circular accelerators, carried out in connection with the design of the CERN alternating gradient proton synchrotron, it was found desirable to have an analogue model simulating these oscillations. Firstly, all analytical methods in use are only approximations whose reliability requires checking. For this an analogue model seemed advantageous as the problems involved are quite formidable and expensive for digital computers. Secondly there are important problems (e.g. in particular the behaviour of oscillations with parameters changing with time) so far more or less inaccessible to analytical methods. These problems can be studied experimentally by means of an analogue model. Experiments of this kind have also proved helpful in the interpretation of analytical results.

So an electromechanical model was devised, simulating the transverse motion of a particle in an accelerator by a two-dimensional pendulum. Non-linear forces and perturbations are simulated by electrostatic fields which, obeying Laplace's equation, make it easy to produce the particular spatial configurations of forces and perturbations occurring in the guide field

accelerators. It was this feature which made a mechanical analogue easier to realize than a purely electronic one<sup>††</sup>.

A short description of the analogue model had been published in the Proceedings of the 1956 CERN Symposium (see Barbier [1956]) together with a first series of experiments carried out mainly to check some of the theoretical results on two-dimensional non-linear resonances. In the present report, details on the performance of the analogue are given as well as results of further experiments on non-linear resonances (with fixed parameters).

It has been found that the constant non-linearity most often applied is the cubic one resulting from an 8-pole field. For this reason it would have been preferable to construct the tube with 8 electrodes rather than with 12, the principal axis of the pendulum being aligned with the axes of two electrodes facing each other.

An excellent 6-pole can be made with such a system of eight electrodes by giving them in

<sup>†</sup> A list of references is given at the end of this paper.

<sup>††</sup> It may be mentioned that another, purely electronic, analogue had been constructed in CERN for the simulation of (one-dimensional) phase oscillation. (See Schmelzer [1956] and Gabillard [1957 and 1958].)

turn the following potentials: 1,  $-\sqrt{2}$ , 0,  $+\sqrt{2}$ ,  $-1$ ,  $+\sqrt{2}$ , 0,  $-\sqrt{2}$ . The voltage differences between adjacent electrodes become smaller with this system for the same multiple component of the potential.

More details and precise calculations for the practical design and construction of the model can be found in a CERN internal report (Barbier, Schoch [1959]).

## 2. The Mathematical Problem to be Simulated by the Model

The mathematical problem to be simulated by the model is given by the equations of motion for the radial and vertical displacements  $x$  and  $y$  in a circular accelerator:

$$\begin{aligned} \frac{d^2x}{d\theta^2} - nx &= -\frac{\partial V(x, y, \theta)}{\partial x} \\ \frac{d^2y}{d\theta^2} + ny &= -\frac{\partial V(x, y, \theta)}{\partial y} \end{aligned} \quad (1)$$

$\theta$  being the angular position of the particle,  $n$  the field index (characterizing the field gradient) and  $V(x, y, \theta)$  a perturbation potential taking account of deviations from the ideal field characterized by  $n$ . This potential satisfies Laplace's equation in two dimensions (if we consider the perturbations due to the guide and focusing field only, which dominate in practice). We write it in the form of a multipole expansion either in polar coordinates  $r, \varphi$  or in rectangular coordinates  $x = r \cos \varphi, y = r \sin \varphi$ :

$$\begin{aligned} V &= \sum_{k=1}^{\infty} V_k \left( \frac{r}{R} \right)^k \cos k(\varphi - \varepsilon_k) \\ &= \frac{V_1}{R} [x + \varepsilon_1 y] + \frac{V_2}{R^2} [x^2 - y^2 + 2\varepsilon_2 xy] \\ &\quad + \frac{V_3}{R^3} [x^3 - 3xy^2 + 3\varepsilon_3 (3x^2y - y^3)] \\ &\quad + \frac{V_4}{R^4} [x^4 - 6x^2y^2 + y^4 + 4\varepsilon_4 (4x^3y - 4xy^3)] + \dots \\ &= \left[ r_0 \left( \frac{\delta B}{B_0} - \frac{\delta p}{p_0} \right) + n\zeta \right] x + [\varepsilon r_0 - n\zeta] y \\ &\quad - \frac{1}{2} [\delta n (x^2 - y^2) + 2\varepsilon n xy] \\ &\quad - \frac{1}{3!} \frac{dn}{dx} [x^3 - 3xy^2 + 3\varepsilon (3x^2y - y^3)] \\ &\quad - \frac{1}{4!} \frac{d^2n}{dx^2} [x^4 - 6x^2y^2 + y^4 + 4\varepsilon (4x^3y - 4xy^3)] + \dots \end{aligned} \quad (2)$$

$R$  is a reference distance from the centre to be chosen later.  $\varepsilon_k$  is the angle by which the plane of symmetry of the  $k$ -th multipole ( $2k$  — pole) is tilted against  $y = 0$ . The last of the equations expresses the multipole coefficient  $V_k$  in terms of accelerator parameters:  $r_0$  is the radius of the equilibrium orbit  $\delta B(\theta)$ ,  $\delta p$ ,  $\delta n(\theta)$  are deviations of field  $B$ , momentum  $p$ , and  $n$ -value from the ideal values;  $\xi(\theta)$ ,  $\zeta(\theta)$  are radial and vertical magnet displacements,  $\varepsilon(\theta)$  is the angle of tilt of the magnetic symmetry plane against  $y = 0$ . The radial derivatives  $dn/dx$ ,  $d^2n/dx^2, \dots$  (taken at  $x = y = 0$ ) are responsible for non-linear terms in the equations of motion ( $dn/dx$  producing a sextupole contribution  $V_3$ ,  $d^2n/dx^2$  an octupole contribution  $V_4$  etc.).

Whereas  $n$  is constant, independent of  $\theta$ , in a weakly focusing accelerator,  $n$  changes with  $\theta$  periodically in an A.G. synchrotron, covering several periods per revolution. In order to simplify the realization of a model, the alternation of  $n$  was not simulated, i.e. the model was designed to perform the "smooth" part of the motion only. To explain what this means we recall that the undisturbed motion ( $V \equiv 0$ ) in the A.G. synchrotron can be represented by the Floquet form of solution:

$$x = \sqrt{a} \left[ w(\theta) e^{i(Q\theta + \varphi)} + \bar{w}^*(\theta) e^{-i(Q\theta + \varphi)} \right] \quad (3)$$

where  $\sqrt{a}$  and  $\varphi$  are arbitrary constants defining amplitude and phase. The factor  $w(\theta)$  is a function of  $\theta$  having the same period as  $n(\theta)$ . Thus (3) is a "smooth" sine motion with frequency  $Q$ , modulated by wriggles with the period of  $n(\theta)$ .  $Q$  as well as the modulating factor  $w(\theta)$  are determined by the shape of  $n(\theta)$ . Under the conditions applying in A.G. synchrotrons, the wriggle modulation is relatively small (about 15% of the sine wave part in the CERN PS). Properly defined "smooth motion" coordinates  $\bar{x}$ ,  $\bar{y}$  can then be shown to satisfy the following equations to a fair approximation (see Schoch [1957], in particular Appendix V).

$$\frac{d^2\bar{x}}{d\theta^2} + Q^2\bar{x} = -\frac{\partial \bar{V}}{\partial \bar{x}}; \quad \frac{d^2\bar{y}}{d\theta^2} + Q^2\bar{y} = -\frac{\partial \bar{V}}{\partial \bar{y}} \quad (4)$$

The unperturbed system can therefore be simulated by a simple two-dimensional harmonic oscillator. The perturbation potential in smooth motion coordinates becomes

$$\begin{aligned}\bar{V} = & \left[ r_0 \left( \frac{\partial B}{\partial B_0} - \frac{\partial p}{\partial p_0} \right) - Q^2 \bar{\xi} \right] \bar{x} + [\epsilon r_0 - Q^2 \bar{\zeta}] \bar{y} \\ & - \frac{1}{2} [\bar{\delta n} (\bar{x}^2 - \bar{y}^2) + 2\bar{\epsilon} Q^2 \bar{x} \bar{y}] - \frac{1}{3!} \frac{d\bar{n}}{d\bar{x}} (\bar{x}^3 - 3\bar{x} \bar{y}^2) \\ & - \frac{1}{4!} \frac{d^2 \bar{n}}{d\bar{x}^2} (\bar{x}^4 - 6\bar{x}^2 \bar{y}^2 + \bar{y}^4) - \dots\end{aligned}$$

In this, the quantities  $\bar{\xi}$ ,  $\bar{\zeta}$ ,  $\bar{\epsilon}$ ,  $\bar{\delta n}$ ,  $d\bar{n}/d\bar{x}$  etc. are modified with respect to those in (2) by the transformation to smooth motion coordinates (see above reference). In general these modifications are small enough to preserve the orders of magnitude of the multipole terms in (2).

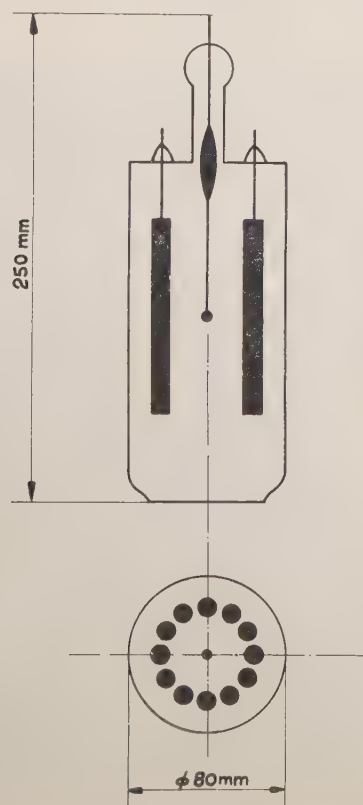


Fig. 1. General layout of the tube.

The conversion from smooth motion back to the complete motion in the a.g. case can be obtained from

$$x = \sqrt{\frac{Q}{2}} (w + \bar{w}^*) \bar{x} - \frac{i (w - \bar{w}^*)}{\sqrt{2Q}} \frac{d\bar{x}}{d\bar{\theta}}, \quad (5)$$

again to a fairly good approximation (formulae (8.8) and (V.2) of above reference).

### 3. Description of the Analogue Model

The model consists of a flexible quartz pendulum in the form of a thin fibre terminating in a ball. The fibre and the ball are made with a piece of fused quartz stretched in a flame. A thin layer of metal is then deposited on the quartz by sputtering in vacuum, thus enabling the ball to carry an electrical charge when the end of the fibre is connected to a voltage source.

The pendulum, being elastic, has its own restoring force, linearly proportional to its displacement from the position of equilibrium. As the displacements occurring within the tube are small in comparison with the length of the pendulum, the linearity of the latter is very good.

The general lay-out is shown in fig. 1. The ball and the fibre are held in the centre of a set of

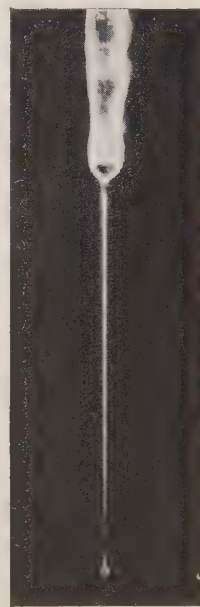


Fig. 2. Photograph of the pendulum.

twelve cylindrical electrodes, arranged in a circle parallel to the fibre axis. These electrodes are made of glass covered with aquadag and can

be connected to voltage sources. The whole is put in a glass tube, which is heated in a furnace and pumped until the vacuum reaches  $10^{-6}$  mm Hg.

The natural damping of the pendulum is then so small that it takes several hours before the amplitude is reduced by a factor  $e$ .

Figs. 2 and 3 are photographs of the pendulum and the tube. Fig. 4 is a complete view of the set-up, showing how the illuminated pendulum can be observed in projection on a screen.

The electrodes are arranged so that the poten-

$$U(r, \varphi, z) = \sum_k U_k(t) \left( \frac{r}{R} \right)^k \cos k(\varphi - \varepsilon_k)$$

where  $r$ ,  $\varphi$  and  $z$  are the co-ordinates,  $R$  the radius of the cylinder through the electrodes and  $U_n$  the voltage harmonics (multipole components) of the field.

The equations of motion of the pendulum therefore become:

$$\frac{d^2x}{dt^2} + \omega^2 x = -\frac{q}{m} \frac{\partial U}{\partial x}; \quad \frac{d^2y}{dt^2} + \omega^2 y = -\frac{q}{m} \frac{\partial U}{\partial y} \quad (6)$$

where  $q$  is the charge,  $m$  the mass, and  $\omega/2\pi$  the natural frequency of the pendulum. Comparison of (2) and (6) shows how the quantities have to be scaled to represent a given system to be simulated.

The pendulum, practically linear and undamped if left alone, can (a) be made non-linear by applying non-linear electrical forces, which

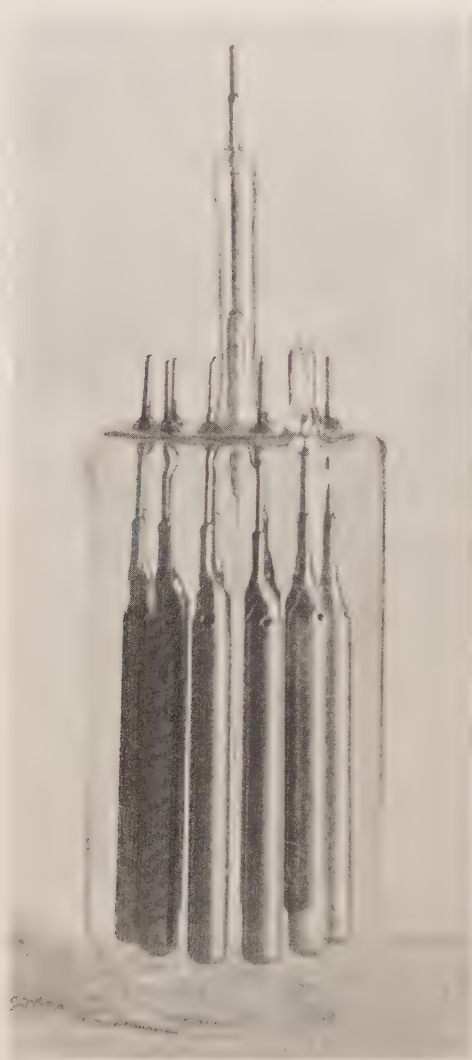


Fig. 3. Photograph of the tube.

tials they create are almost exactly the two-dimensional solutions of Laplace's equation, i.e. they have the form

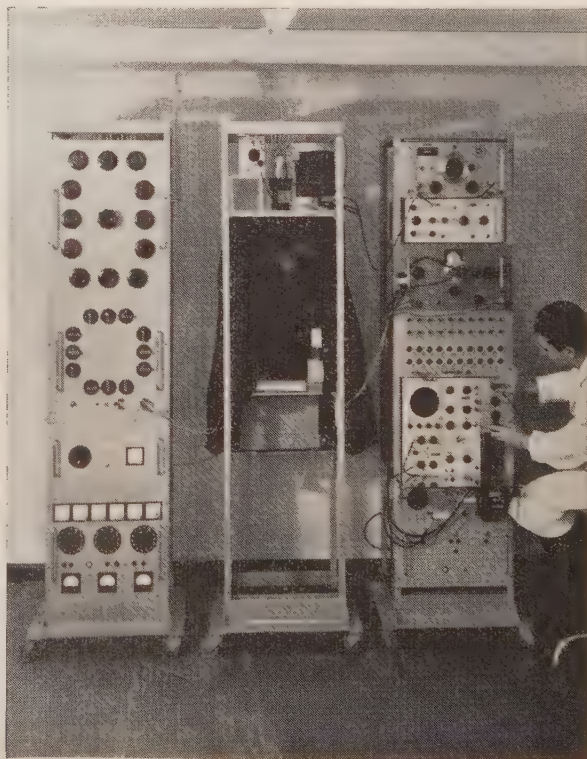


Fig. 4. Photograph of the analogue model.

are constant in time, and (b) be perturbed by time-dependent forces that can also be made non-linear with respect to displacement.

#### 4. Measurements of the Model's Parameters

For the use of the model as a computer, it is preferable to obtain the parameters of the model from measurements taken during its performance, involving only time (or frequencies) and lengths. For instance we calibrate the applied fixed non-linearities by the frequency shift they cause, and the alternating perturbation voltages by the rate of rise of oscillation amplitude with time; in this manner we can treat any mathematical problem directly, i.e. eliminating model constants and scaling factors from a particle accelerator to a betatron analogue. This will allow us to use the model directly as a mathematical instrument for all non-linear oscillation problems.

##### 4.1. THE ONE-DIMENSIONAL MOTION OF THE PENDULUM

##### 4.1.1. Behaviour of the pendulum if no tensions are applied to the electrodes

First the uncharged mechanical pendulum has to be examined. Satisfactory results have been obtained with a pendulum having the dimensions; length 1 cm, thickness 0.1 mm and the volume of the ball 1 mm<sup>3</sup>. The fundamental frequency of the oscillation of the particular tube we used for all experiments described in the following was  $f = 13.3$  c/s. The first higher mode of resonance (flexural oscillations) of the pendulum lies around 110 c/s, this being well outside the range we will use. Such a fiber cannot be made with exact rotational symmetry, so the pendulum has two preferred axes of oscillation, which are perpendicular to each other, and along which it will swing with two slightly different frequencies  $f_x$  and  $f_y$ .

The frequency difference can easily be found by measuring (with a stopwatch) the time  $T$  taken to describe a full cycle of change of the Lissajous pattern in the free oscillation. Then

$$|f_x - f_y| = \frac{1}{T},$$

where  $T = 1.7$  sec for the particular tube used here. Therefore the relative frequency difference is of the order of a few per cent.

$$\frac{f_x - f_y}{f_0} = 0.044.$$

Each of the eigen-frequencies  $f_x + f_y$ , can also be measured separately by comparison with the frequency  $f_g$  of some electronic generator. For this measurement it is convenient to derive an electrical signal which has the exact frequency of the oscillating pendulum or a multiple of it. This can be done by means of a photocell covered by a slit, which produces an electrical pulse each time the image of the illuminated ball passes over it, i.e. twice in a period.

This pulse can be used directly for frequency comparison on a two-beam oscilloscope, when the other beam (from the electronic generator) shows pulses of known frequency; or it can be transformed into a sinusoidal current by means of a selective amplifier which singles out one of the harmonics. If the two pulse rows are used, and remain still with respect to each other, then the frequencies  $f_1$  and  $f_2$  are rational in a ratio  $k_1:k_2$ . This value can be found from the number of pulses  $k_1$  and  $k_2$  in each row, between two coincidences, i.e. for the common period.

If the frequencies differ slightly there will be a drift of one row with respect to the other, and the deviation  $\delta f$  in frequency of the row which is drifting is given by

$$\delta f \cdot \tau = 1,$$

where  $\tau$  is the time taken by one pulse to replace the preceding one with respect to a reference pulse of the other row.

In the case where sine waves are derived from the electronic generator, as well as from the pick-up system, and displayed as a Lissajous pattern on the oscilloscope, the interpretation in terms of frequency differences requires more care, both signals being displayed in such a manner as to be no longer separated. When the pattern is stationary both frequencies remain in an exact rational relationship recognizable by the Lissajous pattern:

$$\frac{f_1}{f_2} = \frac{k_1}{k_2}.$$

To a given frequency ratio still a continuous set of patterns exists, corresponding to the differ-

ences of phase. Therefore, if one of the frequencies,  $f_1$  say, differs from the rational value by a small amount  $\delta f_1$ , the pattern changes continuously and repeats after a time  $\tau$  related to  $\delta f_1$  by

$$\delta f_1 = \frac{1}{k_2 \tau} \text{ c/s.}$$

Similarly, if  $f_2$  deviates,

$$\delta f_2 = \frac{1}{k_1 \tau}.$$

In the above formulae  $k_1$  and  $k_2$  are prime to each other.

The elastic restoring force of the fibre may not be exactly proportional to displacement, the effect becoming apparent when the oscillations reach a certain amplitude.

A non-linear restoring force makes the frequency of the pendulum depend on the amplitude of its oscillation. To a first order approximation the frequency deviation is proportional to the square of the amplitude, if the law of force (with displacement) contains terms of the second or third degree.

With the method of frequency measurement by comparison with an electronic oscillator, quadratic and cubic non-linear forces may be calibrated by measuring the curve of the frequency shift against the square of the amplitude of oscillation, for a time independent non-linearity and one-dimensional free oscillations.

We will examine now what happens, when we place a charge on the pendulum, by connecting its lead to a constant voltage source.

When the ball is thus electrically charged, as in normal operation, the image force, exerted on this charge by the walls of the chamber, i.e. the conducting electrode system, is a negative restoring force which is zero when the ball is in the centre and has linear and non-linear terms depending on displacement.

The linear term of the image force causes a shift of the oscillation frequency of the pendulum, which is independent of amplitude. The non-linear terms of the image force cause a further shift, which grows with amplitude. It

will be seen later that the non-linearity resulting from the image force induced by the electrical charge on the ball is small compared with the non-linearity applied to the model by means of the electrode system in normal operation.

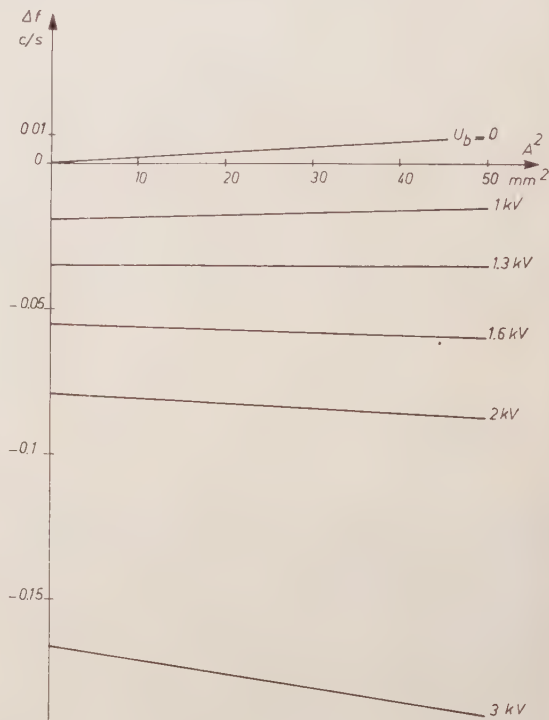


Fig. 5. Frequency shift against the oscillation amplitude squared for various ball voltages.

Fig. 5 shows measured frequency shifts vs. square of free oscillation amplitude for the pendulum, in free motion, and for different voltages on the ball, all electrodes being earthed. As is seen the pendulum has a small inherent non-linearity causing a frequency increase of 0.01 c/s when the amplitude in the tube has reached about 7 mm (the maximum we shall use). As ball voltage is applied one sees that the oscillation frequency is lowered, and that as the oscillation amplitude is increased the frequency still decreases. The charge on the ball introduces a non-linearity which, for the pendulum used, is of opposite sign to the natural one. Both effects cancel out for a voltage of about 1.3 kV on the ball.

Fig. 6 shows the change in frequency for small oscillations vs. ball voltage, i.e. the term in the image force which is linear with displacement.

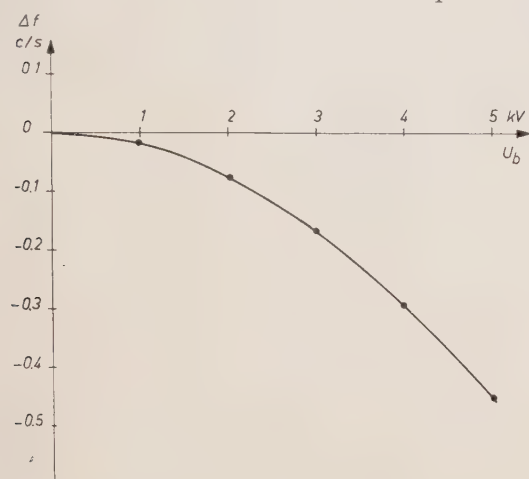


Fig. 6. Frequency change of small oscillations against ball voltage.

The dependence on ball voltage appears to be quadratic, which is to be expected as image forces always depend on the square of the charge involved.

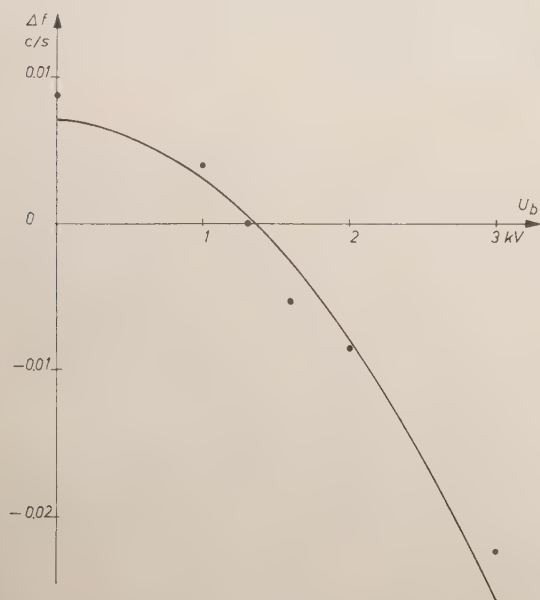


Fig. 7. Frequency shift at an amplitude of 7 mm against ball voltage.

Fig. 7 shows the difference in frequency for maximum (7 mm) and nearly zero amplitudes vs. ball voltage, i.e. the term in the restoring

force which is non-linear with displacement. The observed dependence is also quadratic with ball voltage.

#### 4.1.2. Behaviour of the pendulum on application of time independent voltages to the electrodes, so as to form dipole, quadrupole, sextupole and octupole fields

The pendulum is arranged in the tube so that the principal axis of oscillation falls nearly in the middle of a gap between two electrodes, for symmetry and convenience in experiments on one dimensional oscillations. To produce an electrical dipole field with this system, which has twelve electrodes, four electrodes are connected to the voltage source and two to earth on each side. In this way, the elements in higher order field harmonics are very few, as even harmonics do not appear for symmetry reasons and as the third one vanishes in such an arrangement, where one third of the electrodes in a period are earthed.

Fig. 8 shows the displacements of the ball in a dipole field plotted against the voltage applied to one of the groups of four rods against the earth for various ball voltages against earth. The movement of the ball is seen to be linear with field strength except when very high dipole fields and small ball voltages are used. The effect

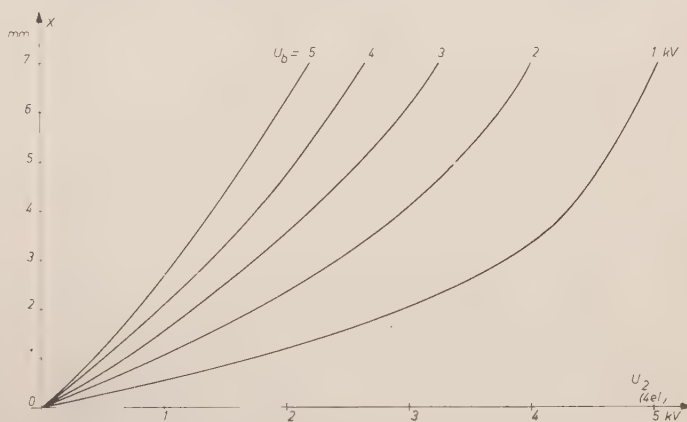


Fig. 8. Displacement of the ball against dipole voltage for various voltages of the ball.

seems to be due to the image force exerted by the charge resting on the electrode system, which is reflecting itself in the conducting surface of the ball.

A quadrupole field is equally well produced with a small content in higher field harmonics by earthing one and connecting two electrodes to the voltage appropriate for each quadrant of the tube. The effect of a quadrupole field with

the amplitude, which is demonstrated in fig. 10 for various six-pole and ball voltages.

Fig. 11 shows the frequency shift obtained with an amplitude of 7 mm for a six-pole versus the product of the ball and the six-pole voltages.

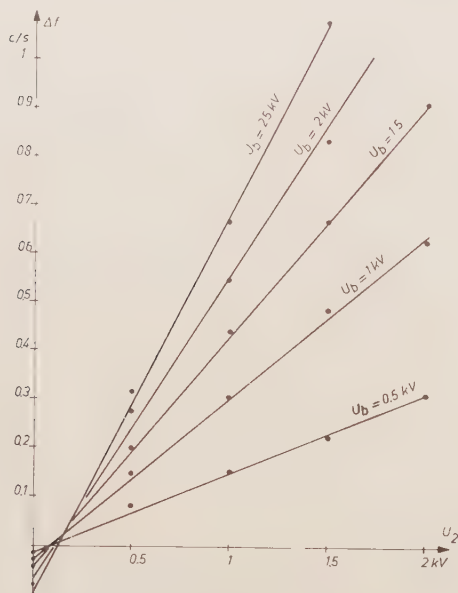


Fig. 9a

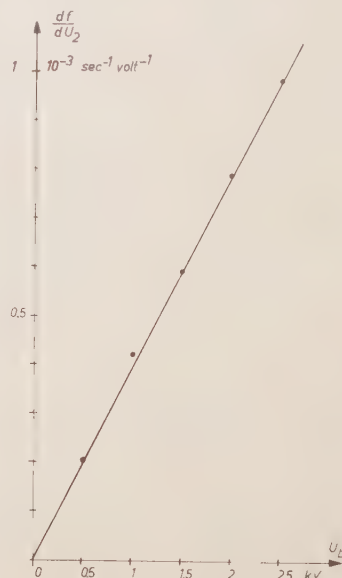


Fig. 9b

Fig. 9. (a) Change of frequency of small oscillations against applied quadrupole voltage, (b) Change of frequency per unit quadrupole voltage as a function of ball voltage.

axes of force in line with the principal oscillation axes is to introduce a new difference between the frequencies of oscillation along the two principal axes, one being increased and the other decreased by equal amounts.

Fig. 9a shows the measured changes in frequency of small oscillations dependent on electrode voltage along one of the principal axes for various voltages on the ball.

Fig. 9b shows how linear the frequency shift is with ball voltage per applied unit quadrupole voltage. A six-pole field is produced by connecting the electrodes in pairs to the voltage sources of opposite polarity. Field components of higher order, owing to their high harmonic number, are probably irrelevant in the centre region of the tube, where the ball is oscillating.

The effect of a six-pole field is to introduce an oscillation frequency dependent on the square of

The curve is nearly parabolic save for the effect of the natural non-linearity which is only noticeable near the origin. The parabolic dependence of frequency shift on six-pole strength must be expected from theoretical considerations whereas in the eight-pole case the dependence will be linear, as will be confirmed by experimental work.

An eight-pole field may be produced in the following way. Take a system, connect one electrode to a voltage source and two electrodes to another source of opposite polarity and of half the magnitude.

As in the six-pole case, the eight-pole introduces a linear dependence of oscillation frequency on the square of the amplitude as is shown in fig. 12 for various ball and eight-pole voltages. The frequency differences obtained are proportional to the applied eight-pole force as shown in

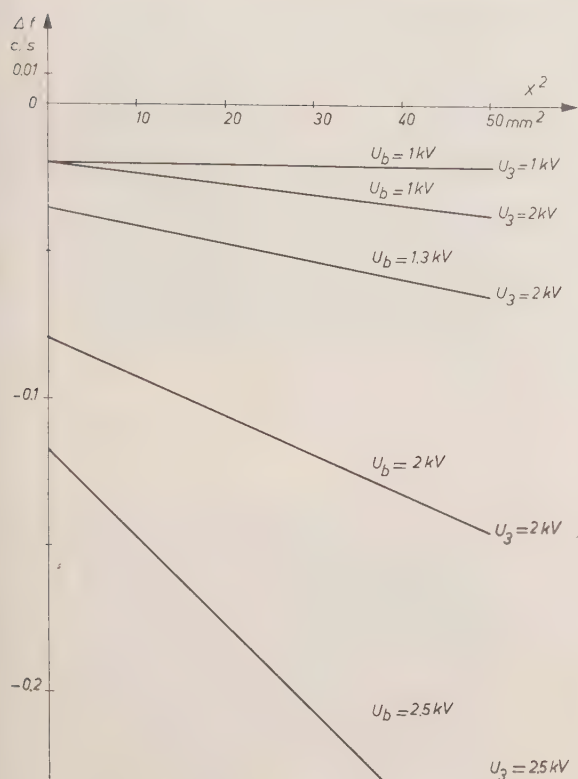


Fig. 10. Frequency shift against the oscillation amplitude squared for various six-pole and ball voltages.

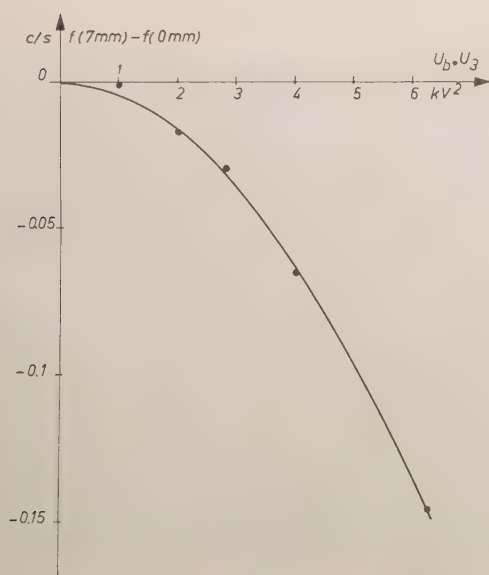


Fig. 11. Frequency shift at an amplitude of 7 mm for a six-pole against the product of six-pole and ball voltages.

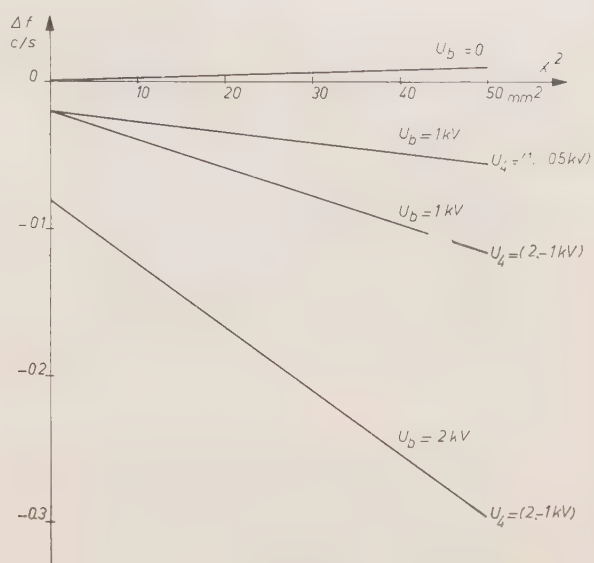


Fig. 12. Frequency shift against the oscillation amplitude squared for various eight-pole and ball voltages.

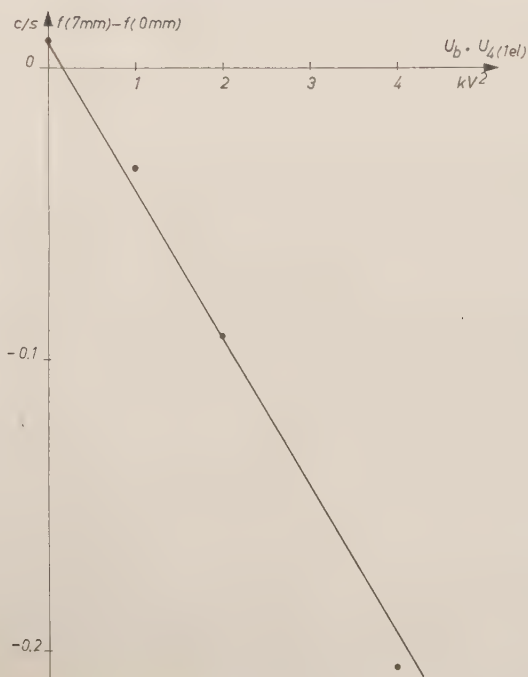


Fig. 13. Frequency shift at an amplitude of 7 mm for an eight-pole against the product of eight-pole and ball voltages.

fig. 13 where the frequency differences for maximum amplitude are drawn as a function of the product of the eight-pole voltage and the ball voltage.

#### 4.2. THE TWO-DIMENSIONAL MOTION OF THE PENDULUM

##### 4.2.1. *Natural two-dimensional oscillations of the uncharged pendulum*

If the oscillation frequencies of the ball along each principal axis were the same, the motion of the ball would describe an ellipse. As both frequencies differ slightly, the phase between the oscillations along the  $x$  and  $y$  axes changes slowly, thus altering the shape of the ellipse. The ellipse, however, should remain always inscribed in a rectangle, when the pendulum is perfectly linear. If the pendulum is subject to non-linear forces the figure in which the ellipses are described takes a non-rectangular form.

Fig. 14a is a photograph showing a free two-dimensional oscillation of the ball, with no voltages applied whatsoever. Owing to the non-rectangular shape of the envelope, one must conclude that there is a structural non-linearity in the pendulum.

Fig. 14b shows the same free two-dimensional oscillation, this time with an applied ball voltage of 1.3 kV. For this ball voltage the oscillation frequency was found independent of the amplitude, the frequency drift of the image force non-linearity compensating that of the natural non-linearity. As the image force does not differ basically from the preceding one, it is seen that the image force non-linearity cannot compensate the structural one for a two-dimensional motion.

Fig. 14c shows another free two-dimensional motion, with a ball voltage of 3 kV, which makes the image force several times larger. As is seen, the form of the pattern is not substantially altered.

As the image force is a pure radial one, and independent of azimuth, it follows that the structural non-linearity must be an azimuth dependent force.

##### 4.2.2. *Correction of the inherent non-linearity by a static eight-pole field*

An attempt was made to correct the pendulum's non-linearity by applying an external eight-pole. Fig. 15 shows the result for an eight-pole DC voltage of  $-0.5$ ,  $+1$  kV and a ball voltage of 1.5 kV. The pendulum became linear, although its amplitude was very large. The eight-pole field is somewhat larger than expected judging from the one-dimensional frequency shift curves. It has however the correct sign.

##### 4.2.3. *Motion of a linear conical pendulum in six and 8-pole fields*

It was tempting after having produced an ideally two-dimensional pendulum, to examine the form of the oscillation in pure six-pole and eight-pole fields. The corrected eight-pole voltage was maintained, to which one added a strong six-pole or eight-pole DC voltage. The results are shown in figs. 16a and 16b.

In the six-pole case, the envelope has only one axis of symmetry and in the other case it has two perpendicular ones, which is in accordance with the symmetry properties of the applied non-linear fields (i.e. a six-pole and an eight-pole).

#### 5. Behaviour of the Model under Perturbations of Constant Frequency

One will now study the effect of multipole perturbations of fixed frequency on one and two dimensional oscillations: first without and then with constant applied non-linear restoring forces (so called stabilizing non-linearity).

##### 5.1. BEATING OF AMPLITUDES IN ONE DIMENSION

The experiments we will describe now have been carried out using mainly a one-dimensional oscillation. When the exciting multipole fields are arranged to be symmetrical with respect to the main oscillation axis of the pendulum and when the latter performs a one-dimensional oscillation along its axis, (which lies in the direction of the field at the moment when the perturbation is switched on) then the motion remains one-dimensional, even when excited by multipole fields of a higher order than the dipole.

The purpose of the work is to examine the

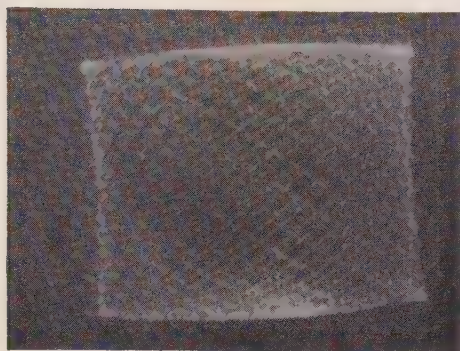


Fig. 14a

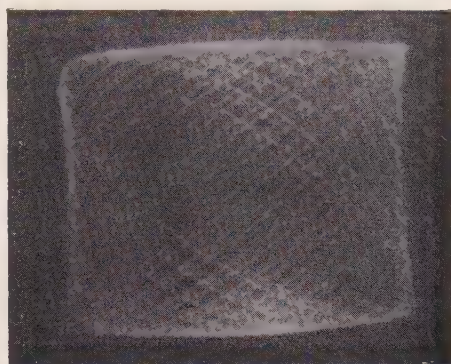


Fig. 14b

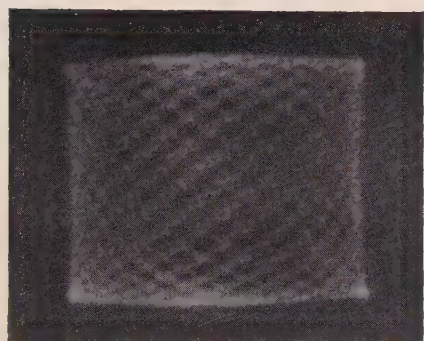


Fig. 14c

Fig. 14. Free two-dimensional oscillation of pendulum (ball voltage, (a) 0, (b) 1.3, (c) 3 kV optical magnification 5).

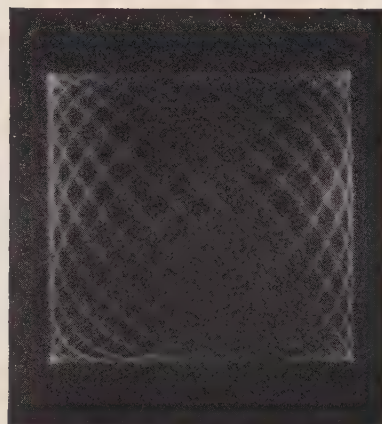


Fig. 15. Two-dimensional oscillation of pendulum when corrected with static eight-pole (optical magnification 5).

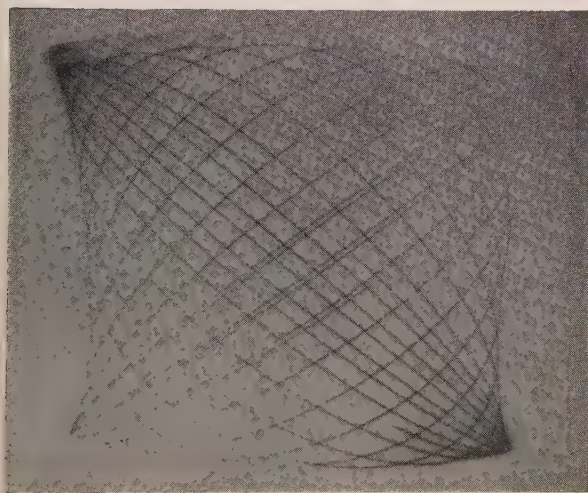


Fig. 16a

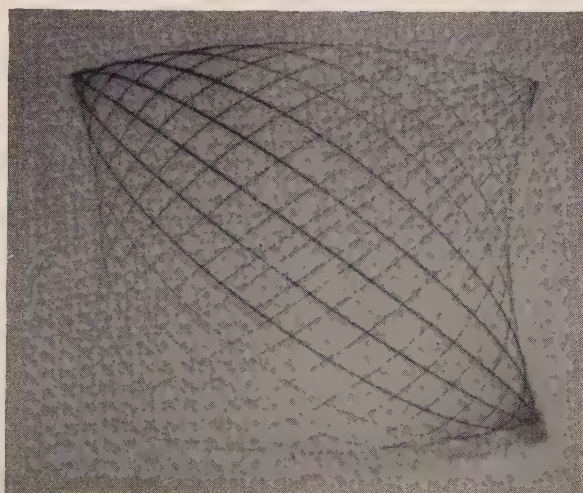


Fig. 16b

Fig. 16. Two-dimensional oscillation of a perfectly linear pendulum, a) in a pure static six-pole field, b) in a pure static eight-pole field.



Fig. 17. View of the tube taken along the axis showing pendulum performing one-dimensional motion.

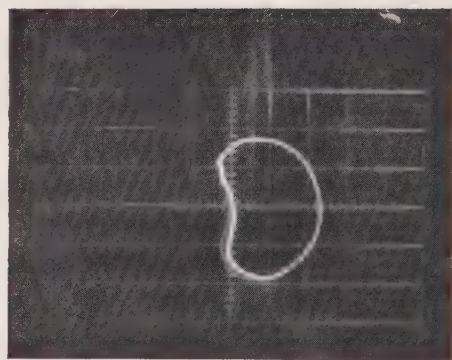


Fig. 19c

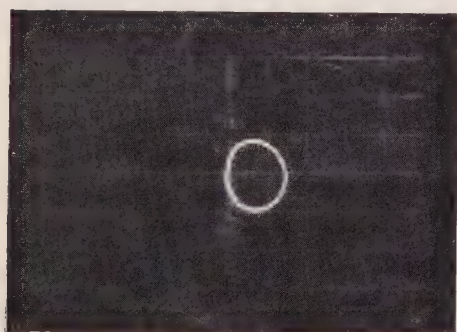


Fig. 19a

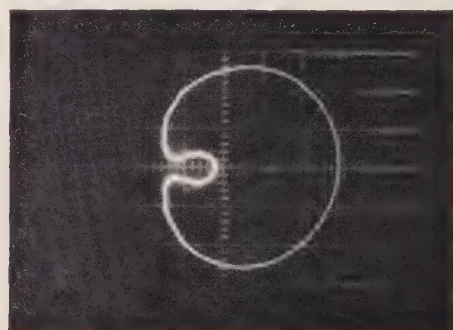


Fig. 19d

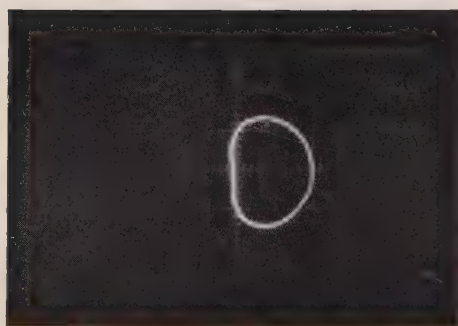


Fig. 19b



Fig. 19e

Fig. 19. Motion in the phase plane for start at zero oscillation amplitudes with dipole excitation and stabilizing non-linearity at several distances from eigen-frequency, (a)  $\Delta f = -0.0244$  c/s, (b)  $\Delta f = 0$ , (c)  $\Delta f = +0.0253$ , (d)  $\Delta f = 0.0485$ , (e)  $\Delta f = 0.0625$ .

behaviour of an anharmonic pendulum, submitted to a periodic force, which is acting in the direction of its line of oscillation and which depends on a power of the displacement. Mathematical treatment of this problem, with special reference to particle accelerators, has been carried out by Judd [1950], Moser [1955], Sturrock [1958], Hagedorn [1957], Hagedorn and Schoch [1957] and Courant [1956].

up system, which provided a current proportional to the displacement of the pendulum. It measured the variation of the capacity between the pendulum and a pair of adjacent electrodes of the model. This capacity was put in parallel with a tank circuit in the screen grid of a tetrode, while a quartz crystal was driving the control grid. The tube was operated on its dynatron characteristic, in order to provide a negative resistance.

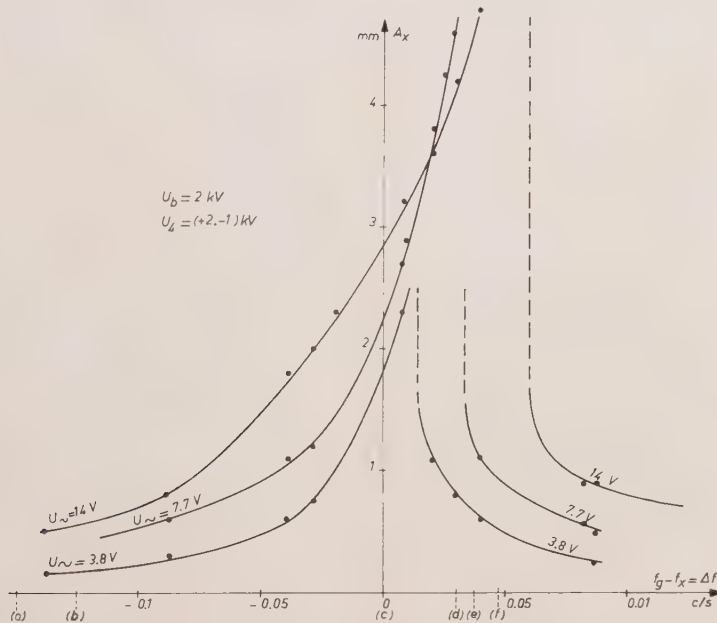


Fig. 18. Resonance curves for one-dimensional oscillation with dipole excitation and eight-pole stabilizing non-linearity.

It was desired to check the above work experimentally in order to justify its application to the designing of the machines.

The pendulum was made anharmonic by the application of a static eight-pole (the so-called stabilizing non-linearity) well lined up along the main axis of oscillation.

Fig. 17 is a photograph showing how well the pendulum can be made to oscillate along a straight line (even at large amplitudes). The scale is represented by the electrodes system.

Excitation voltages were applied in the forms of a dipole, a quadrupole and a six-pole respectively, in order to have the cases where the perturbation force was independent of displacement, proportional to it, and the square of it. For these experiments we had an electrical pick-

up system, which provided a current proportional to the displacement of the pendulum. It measured the variation of the capacity between the pendulum and a pair of adjacent electrodes of the model. This capacity was put in parallel with a tank circuit in the screen grid of a tetrode, while a quartz crystal was driving the control grid. The tube was operated on its dynatron characteristic, in order to provide a negative resistance.

#### 5.1.1. Dipole excitation with stabilizing eight-pole

In this case the excitation field is homogenous i.e. the time varying force is independent of the position of the pendulum. When the excitation frequency is in the neighbourhood of the eigen-

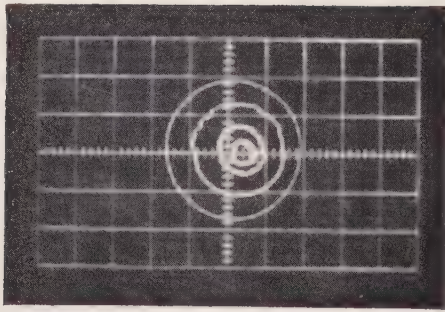


Fig. 20a

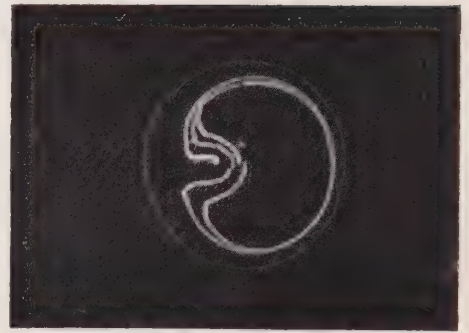


Fig. 20d

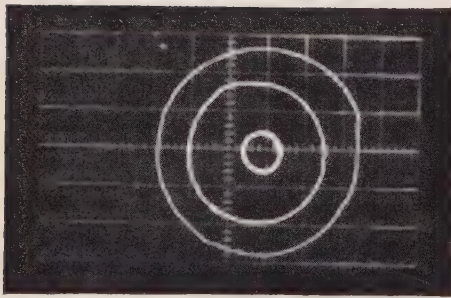


Fig. 20b



Fig. 20e

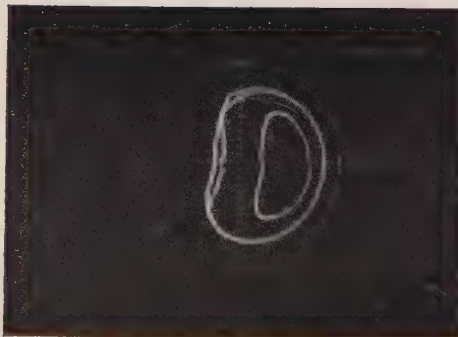


Fig. 20c

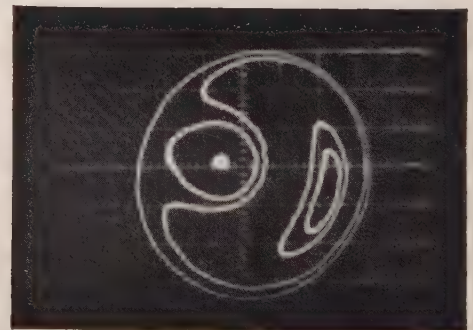


Fig. 20f

Fig. 20. Maps of the phase plane with dipole excitation and stabilizing light-pole at several distances from eigen-frequency, (a)  $\Delta f = -0.015$  c/s, (b)  $\Delta f = -0.012$ , (c)  $\Delta f = 0$ , (d)  $\Delta f = 0.03$ , (e)  $\Delta f = 0.0385$ , (f)  $\Delta f = 0.0485$ .

frequency of the pendulum, it corresponds to an integer resonance line in the machine. This case deserves to be examined in detail, because it shows all the main features of the behaviour of the anharmonic oscillator of a main resonance. It will not be basically different at higher order resonance modes, when the excitation force depends on the position of the pendulum as with a quadrupole or a six-pole.

Fig. 18 shows resonance curves taken point by point. They are obtained by measuring the maximum amplitude of oscillation reached by the pendulum, starting from zero, at different frequencies and for different excitation voltages.

Fig. 19 shows a series of patterns described by the representative point, taken at equal periodic intervals in the phase plane, starting at zero oscillation amplitude and with constant perturbation voltage at different frequencies.

The phase space curves have been taken with the same ball and eight-pole voltages as for the previous figure but with an excitation voltage of 16 volts, i.e. they are referring to a situation almost identical with the one of the upper of the three resonance curves.

It was interesting to examine the motion when the starting perturbation for the initial oscillation amplitude differed from zero, but using the same exciting voltage at the same frequency, in order to get a general view of the patterns described over the phase plane by the representative point.

Fig. 20 shows several such cases taken at various distances from the resonance and shows rather well the way in which these curves divide when passing through resonance.

The voltages used were 1.3 kV on the ball, (1 —, + 2 kV) on the eight-pole and 7.7. volts for excitation i.e. near the second resonance curve in fig. 18. The static displacements of the ball, in a dipole field, could have been taken to calibrate the dipole exciting force; however, it was possible to measure the rate of growth of oscillation amplitude.

Fig. 21 shows the rate of growth of oscillation amplitude at exact resonance frequency for several dipole alternating voltages and ball voltages.

The non-linearity induced by the eight-pole has already been calibrated, by the shift of the oscillation frequency with its amplitude.

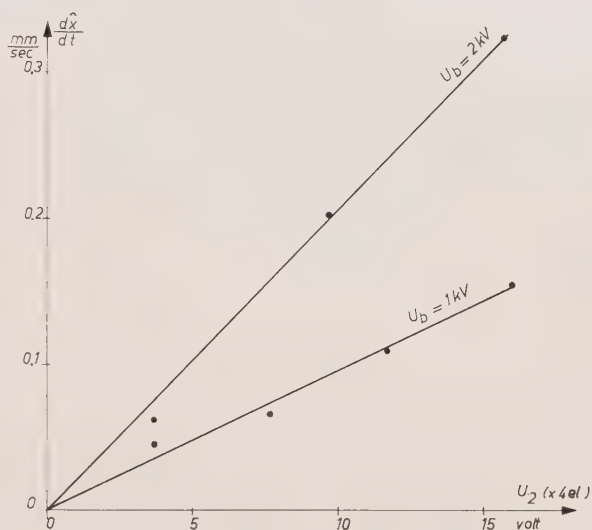


Fig. 21. Rate of growth of oscillation amplitude against exciting dipole voltage.

The eight-pole can be calibrated otherwise, when the dipole excitation and distance from resonance are known, by using the velocity along its trajectory at the representative point in phase space. This can be measured on some of the photographs by counting the number of dots along the trace, the dots appearing in synchronism with the excitation voltage.

#### 5.1.2. Quadrupole excitation of double eigenfrequency with stabilizing eight-pole

The perturbation force is now proportional to the displacement of the ball and its frequency is twice  $f_x$ . This corresponds to being on a half-integer resonance line, for the betatron oscillation in a synchrotron. The motion was again kept one-dimensional.

Fig. 22 shows several diagrams of the traces in the phase plane. The ball voltage was 1.3 kV and the quadrupole excitation voltage, 250 V. Figs. 22a and 22b show the case where the eight-pole is not yet applied; b is taken very near resonance and shows the hyperbolae along which the particle escapes in the phase plane. Fig. 22a, however, is taken rather far from resonance, so that the particle performs a beating.

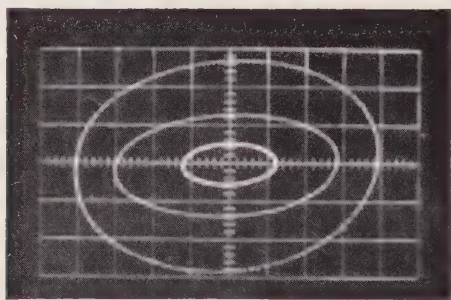


Fig. 22a

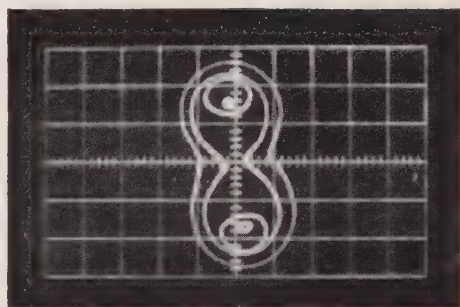


Fig. 22d

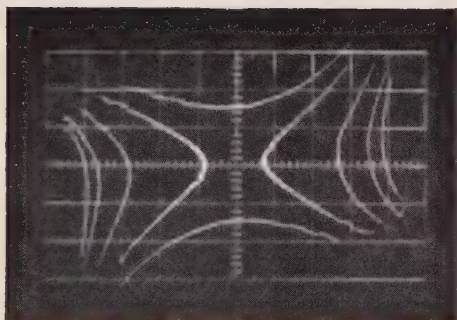


Fig. 22b

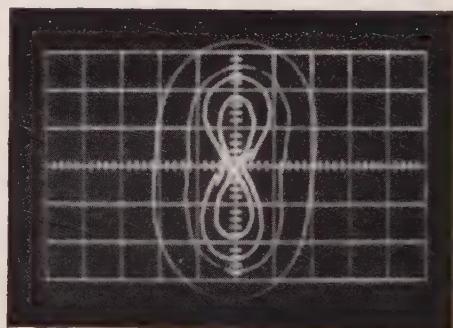


Fig. 22e

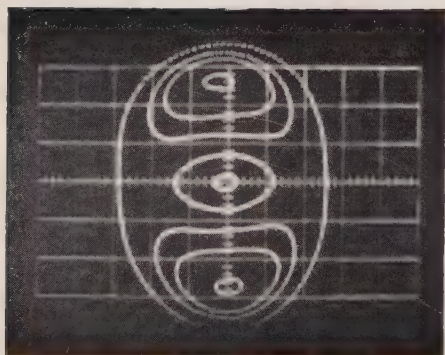


Fig. 22c

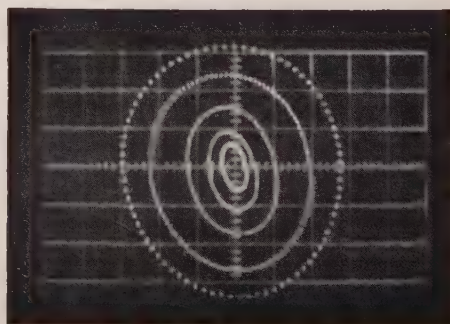


Fig. 22f

Fig. 22. Maps of the phase plane in the case of quadrupole excitation with and without stabilizing eight-pole. Without eight-pole, (a)  $\Delta f = f_x - \frac{1}{2} f_g = -0.055$  c/s, (b)  $\Delta f \gtrsim 0$ . With eight-pole, (c)  $\Delta f = -0.07$  c/s, (d)  $\Delta f \gtrsim 0$ , (e)  $\Delta f = 0$ , (f)  $\Delta f = +0.023$  c/s.

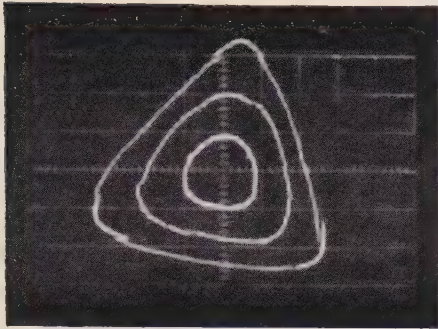


Fig. 23a



Fig. 23d



Fig. 23b



Fig. 23e

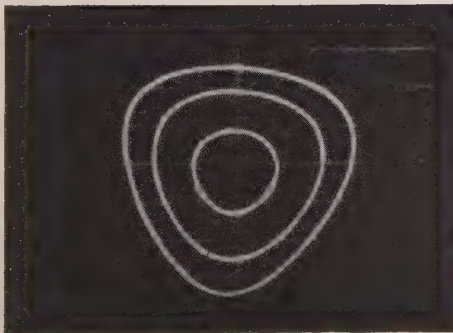


Fig. 23c

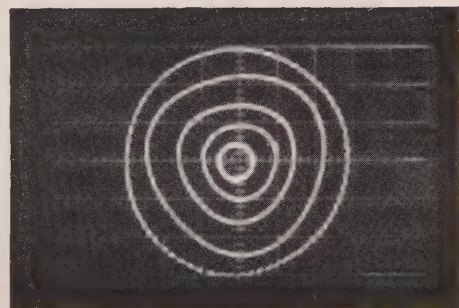


Fig. 23f

Fig. 23. Maps of the phase plane in the case of six-pole excitation with and without static eight-pole. Without eight-pole, (a)  $\Delta f = f_x - \frac{1}{3} f_g = -0.04$ , (b)  $\Delta f \gtrsim 0$ , (c)  $\Delta f = 0.0555$  c/s. With eight-pole, (d)  $\Delta f = -0.0238$ , (e)  $\Delta f \gtrsim 0$ , (f)  $\Delta f = 0.0258$  c/s.

The other diagrams are taken with an applied static eight-pole ( $-1, +2$  kV). One sees how the elliptical but upright beating curves deform to the right, giving rise to a central and two lateral islands of stability.

### 5.1.3. Six-pole excitation of triple eigen-frequency with stabilizing eight-pole

The perturbation force is now proportional to the square of the displacement of the ball and its frequency is about three times  $f_x$ , corresponding to a third order sub-resonance line in a synchrotron. The motion is one dimensional as before.

Fig. 23 shows several traces in the phase plane:  $a, b, c$ , without eight-pole,  $d, e, f$  with eight-pole. Voltages were 1.3 kV on the ball, ( $+1, -0.5$  kV) on the eight-pole and 250 V for excitation.

The phase space diagrams of the three cases resemble closely those calculated analytically (Schoch [1957]). Thus a satisfactory agreement is established between the theoretical and the experimental work.

## 5.2 BEATING OF AMPLITUDES IN TWO DIMENSIONS

In this paragraph we will examine the effect of multipole perturbations of fixed frequency on two-dimensional oscillations. Constant non-linear restoring forces were not applied, except in one particular case, where we wanted to show the form of the oscillation pattern when the resonance was quenched.

### 5.2.1. The resonance lines of the linear conical pendulum in the $f_x, f_y$ plane when subject to multipole perturbations

The perturbing multipole field has a frequency  $f_g$ , given by the electronic generator. It has been found that the pendulum has two different frequencies along its axis of oscillation. These frequencies, which are already different in the natural pendulum, can be made more so by applying a quadrupole force with the electrodes, which is constant with respect to time.

As is known, the effect of such a quadrupole is only to change the difference between the two natural frequencies. Let  $f_x$  and  $f_y$  denote the

actual frequencies, which will have been adjusted within this range, and which can be measured directly with a photocell. We are now looking for the perturbation frequency  $f_g$ , for which the resonances of the pendulum occur with a given couple of pendulum frequencies  $f_x, f_y$ .

It is convenient to plot the resonance points in a two-dimensional coordinate system, as a function of the quantities  $f_x/f_g, f_y/f_g$ . Such a graph (see fig. 24) gives all the required information for the resonance states.

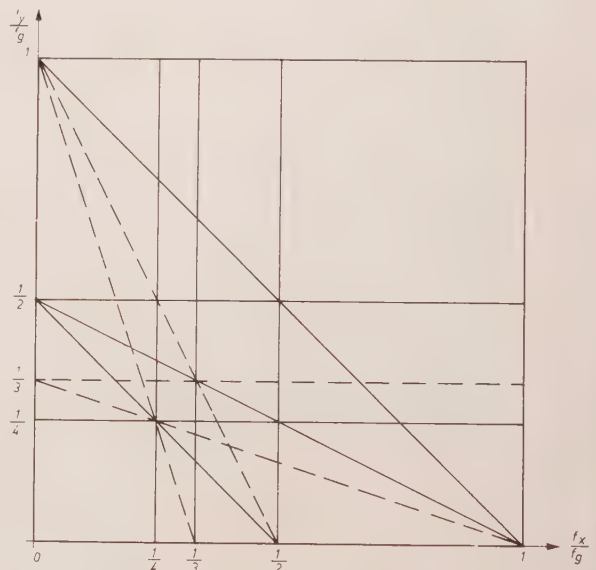


Fig. 24. The frequency plane of the model, with the sum resonance lines.

Using our type of multipole excitation, the resonances are expected to occur (according to Hagedorn) along the bunches of lines which have been drawn in the diagram going through the points  $f_x/f_g = f_y/f_g = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$  for the two, four, six and eight-pole cases respectively. The dotted lines are those not expected to appear when the axis of perturbation ( $\varphi = 0$  in  $\cos k\varphi$ ) coincides with the  $x$ -axis of the pendulum. As it is impractical to make  $f_x$  very different from  $f_y$ , each bunch of resonance lines will be examined experimentally only in the region where all the lines of the bunch have a point in common.

An accurate measurement can be made of a set of frequencies in the region of a nodal point,

with the help of the beating between the oscillation of the pendulum, along one of its axes and the perturbation generator frequency.

In the region of the knot of the resonance lines of order  $n$ , the generator frequency is approximately

$$f_g = nf_x + \delta f_g$$

where  $\delta f_g$  is the small deviation from the measured value of  $nf_x$ . Dividing by  $nf_g$ :

$$\frac{f_x}{f_g} - \frac{1}{n} = -\frac{\delta f_g}{nf_g}$$

giving the coordinate of the point in  $f_x/f_g$  direction. The quantity  $(f_y/f_g) - (1/n)$  can be found in a similar way, or  $f_y$  can be measured directly from  $f_x$  with absence of perturbations, by the period of the Lissajous pattern  $f_x, f_y$  of the pendulum itself

$$|f_x - f_y| = 1/T.$$

It is generally known which of the values  $f_x$  or  $f_y$  is the greater or it may easily be determined by tuning a selective amplifier to these two frequencies. The measured resonance points for sub-resonances of order  $n = 2, 3, 4$  as seen in

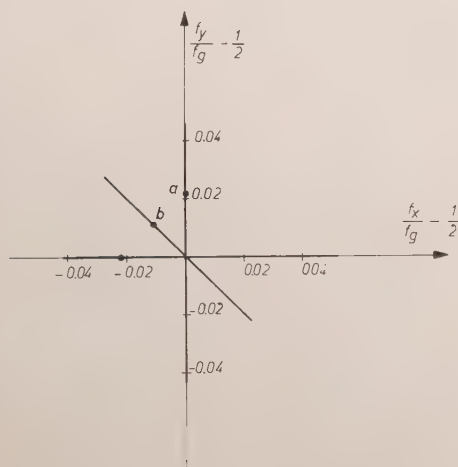


Fig. 25. The observed second order sub-resonances in the frequency plane.

figs. 25, 26 and 27 were found to fit exactly with Hagedorn's theoretical lines and so a few measurements were considered sufficient. It was also made sure that no supplementary lines

could be found in between these lines. The dotted resonance lines did not appear when the  $x$  axis

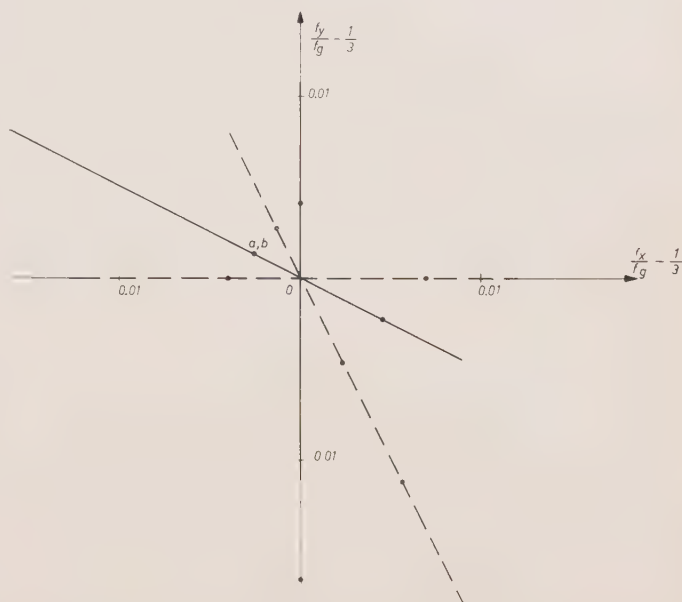


Fig. 26. Observed third order sub-resonances in the frequency plane.

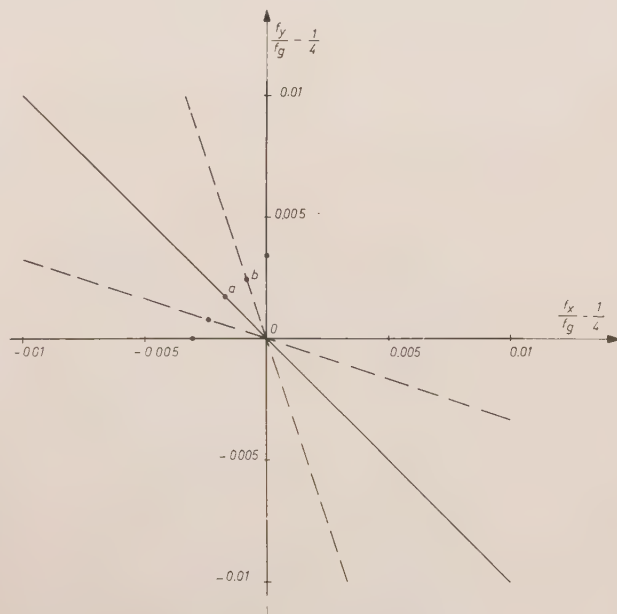


Fig. 27. Observed fourth order sub-resonances in the frequency plane.

of the pendulum was the same as the axis of the multipole perturbation i.e. when it went through

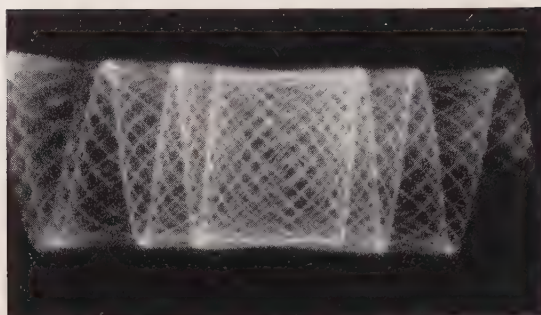


Fig. 28a

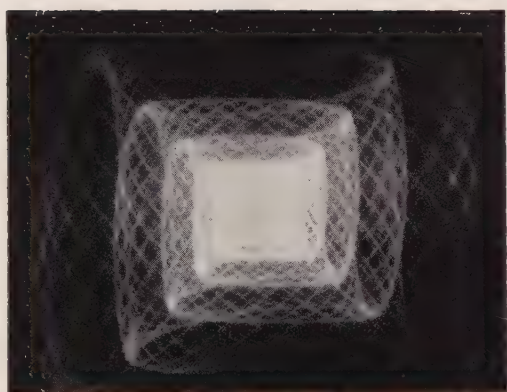


Fig. 28b

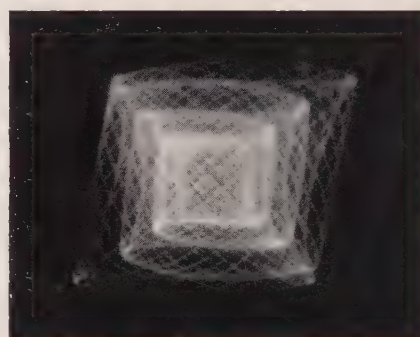


Fig. 28c

Fig. 28. Growth of oscillation amplitude at second order sub-resonance with quadrupole (a)  $2f_y = f_g$ , (b)  $f_x + f_y = f_g$ , (c)  $f_x + f_y = f_g$  with static eight-pole.

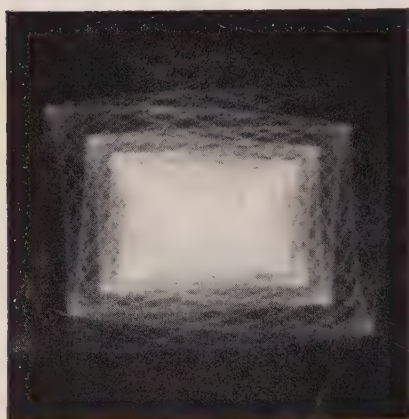


Fig. 29a

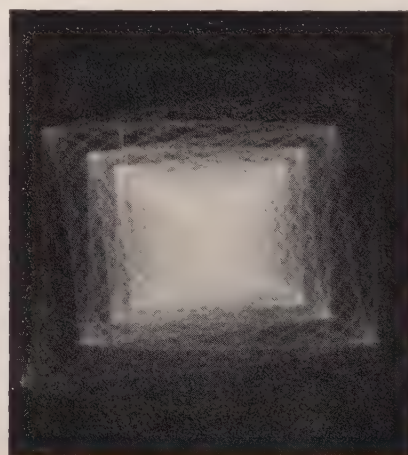


Fig. 29b

Fig. 29. Growth of oscillation amplitudes at third order sub-resonance with six-pole  $f_x + 2f_y = f_g$  and different initial conditions (pendulum oscillating (a) horizontally, (b) vertically).

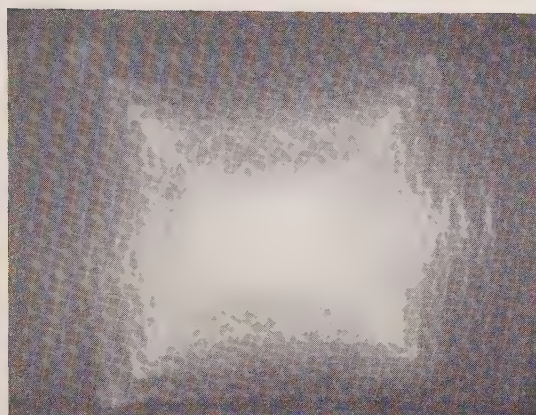


Fig. 30a

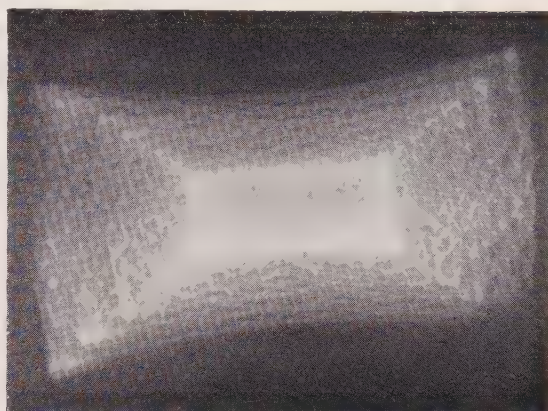


Fig. 30b

Fig. 30. Growth of oscillation amplitude at fourth order sub-resonance (a)  $2f_x + 2f_y = f_g$ , (b)  $3f_x + f_y = f_g$ .

the point of the periphery having the maximum voltage distribution [ $\cos n\varphi = 1$ ]. These lines could be excited only when an angle between the axes was introduced, by shifting the potential distribution of the perturbation around the azimuth. This too agreed with Hagedorn's conclusions.

### 5.2.2. The quadratic invariant of the two-dimensional oscillator subject to multipole perturbations of fixed frequency

The behaviour of the particle itself at sub-resonances is shown in the following photographs taken of the actual motion of the ball.

Fig. 28a shows the growth of oscillation amplitude at a second order sub-resonance with a quadrupole excitation for  $f_g = 2f_x$ . This is the quasi one-dimensional case where the amplitude increases in the  $x$  direction only.

Fig. 28b shows the motion when  $f_g = f_x + f_y$  i.e. using the  $45^\circ$  sub-resonance line. In this case the amplitudes increase in both the  $x$  and  $y$  directions. As the central amplitudes were nearly equal, the corners of the envelopes of the successive Lissajous pattern are seen to lie on two perpendicular straight lines, which are in fact the quadratic invariant for this case where  $n_1 = n_2$ ,  $r_1 = r_2$ .

Fig. 28c shows what happens when a constant eight-pole non-linearity as well as the perturbation is applied to the pendulum. The amplitudes

are limited and the oscillations increase and decrease in a cyclical manner.

Fig. 29 shows the motion of the particle at a 3rd order sub-resonance with a six-pole perturbation on a sloped resonance line  $f_g = f_x + 2f_y$ . In fig. 29a the initial oscillation is directed along the horizontal axis and in fig. 29b it is directed along the vertical axis. The corners of

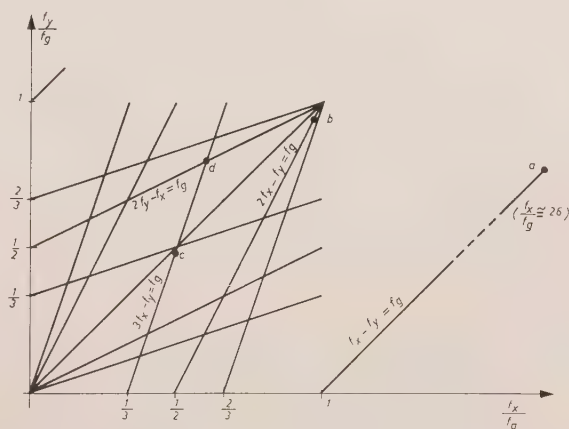


Fig. 31. The difference resonance lines in the frequency plane showing the points, a, b, c and d where the photographs of fig. 32 were taken.

successive envelopes move along hyperbolae, having half axes in the ratio  $\sqrt{2}:1$  (the quadratic invariant for the case where  $n_1 = 1$ ,  $n_2 = 2$ ).

Fig. 30 shows the growth of amplitude with eight-pole perturbation of frequencies (a)  $f_g = 2f_x + 2f_y$  and (b)  $f_g = f_x + 3f_y$ . The quadratic

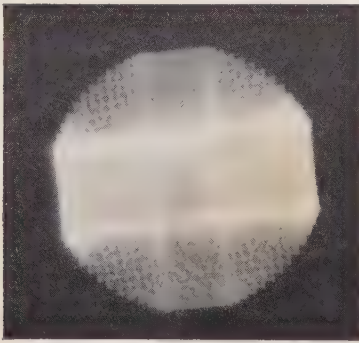


Fig. 32a

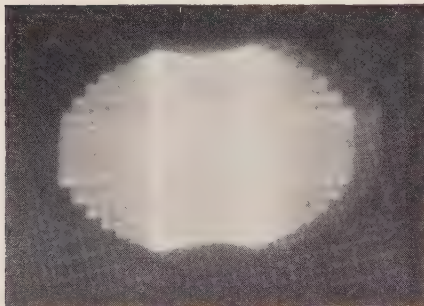


Fig. 32b

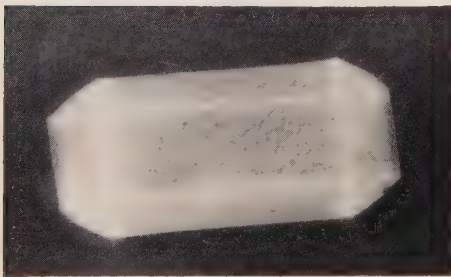


Fig. 32c

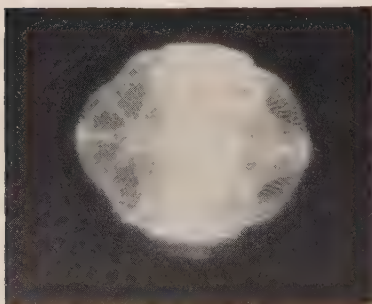


Fig. 32d

Fig. 32. Behaviour of pendulum on difference resonance lines (a) with quadrupole,  $f_x - f_y = f_g$ , (b) with six-pole,  $2f_x - f_y = f_g$ , (c) with eight-pole,  $3f_x - f_y = f_g$ , (d) with six-pole,  $2f_y - f_x = f_g$  and  $f_y/f_x = 4/3$ .

invariants are for (a) an equilateral hyperbola (b) a hyperbola with axes in the ratio  $\sqrt{3}:1$  and this is fairly well shown by the photographs.

The cases where the quadratic invariant is an ellipse are shown next. We wished to ascertain in the above cases that the effect of the so-called difference resonance lines is not causing particle loss for not too large amplitudes.

Fig. 31 shows frequency diagrams with some possible difference resonance lines drawn.

Fig. 32 shows photographs taken at certain points indicated in fig. 31. In fig. 32 the motion of the pendulum is shown when excited

(a) with a quadrupole and a frequency

$$f_g = f_x - f_y, \quad \text{for} \quad f_x \approx f_y$$

(b) with a six-pole and a frequency

$$f_g = 2f_x - f_y, \quad \text{for} \quad f_x \approx f_y$$

(c) with an eight-pole and a frequency

$$f_g = 3f_x - f_y, \quad \text{for} \quad f_x \approx f_y$$

(d) with a six-pole and a frequency

$$f_g = 2f_y - f_x, \quad \text{for} \quad f_y/f_x = 4/3.$$

The envelope of all the oscillations is seen to be in (a) a circle (b) an ellipse with the ratio of the axes equal to  $\sqrt{2}$  and in (c) an ellipse with the ratio of the axes equal to  $\sqrt{3}$ , as was expected. The ratio for (d) is 1.2 instead  $\sqrt{2}$ .

It is probable that the oscillations do not cover the whole circle or ellipse for the following reasons: inadequacy of exciting voltage, inexact timing or drift, presence of natural or electric non-linear forces, which exert a limiting action on the amplitude.

The behaviour of the model under perturbations of a slowly drifting frequency has been studied for one-dimensional motion in the case of the main resonance by Keller, Fidecaro and Barbier [1957] and in more detail by Robouch [1959]. Further plans for work on the model include sweeps through sub-resonances.

### Acknowledgements

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† Unpublished but available for consultation at main accelerator centres.

## THE ELECTRON VAN DE GRAEFF IN SEMICONDUCTOR RESEARCH

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Received 3 August 1959

A 1 MeV electron Van de Graaff accelerator has been applied to fundamental semiconductor research. Its versatility as a pulsed source of ionization for studying electrical transients in these materials and under d-c

operation for introducing atomic defects into solid lattices has been exploited. These two aspects of the accelerator are discussed with examples from experiments in germanium.

## 1. The Van de Graaff Apparatus

A small electron Van de Graaff accelerator has been used at Bell Telephone Laboratories in connection with a program of research on semiconductors. Some of the aspects of its use are rather specific to the field of semiconductors while others have applicability to metals and insulators as well. Attention will be drawn to these similarities and differences in two places.

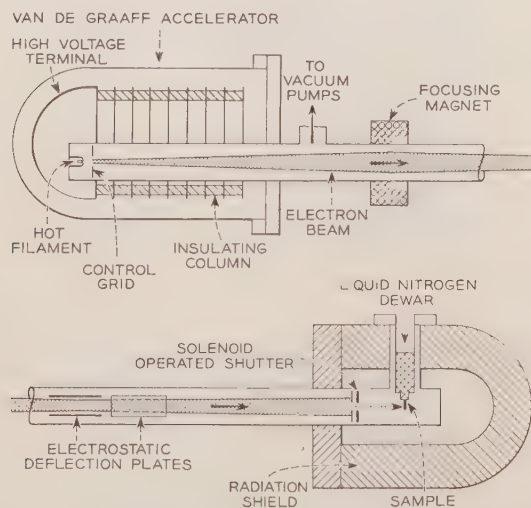


Fig. 1. Schematic diagram of the Van de Graaff radiation facility.

Fig. 1 is a schematic diagram of the arrangement of the horizontal Van de Graaff and its associated apparatus. At the upper left in the figure is a diagram of the accelerator itself. This is an electron machine capable of 1 MeV at a d-c current of 25 microamperes. After accelera-

tion the electrons pass through a focusing magnet and then through two pairs of electrostatic deflection plates. The deflection plates eliminate the necessity of using scattering foils for uniform exposure of a sample area—a feature which is significant for some problems; for example, precise energy work. These plates do not give very large deflections to the beam. At 1 MeV only about 1 centimeter in the horizontal or vertical directions can be covered. This area is, however, quite adequate for the samples used. The two sets of plates are provided with saw tooth voltages the order of a kilovolt in magnitude at incommensurate frequencies. They are also provided with d-c potentials to permit fine positioning adjustment of the beam. The beam enters a radiation shield at the end of the focusing and deflecting space. It may be stopped by a solenoid operated shutter. Beyond this, the beam impinges on a sample which is supported on the bottom end of a dewar containing liquid nitrogen or some other appropriate cooling liquid. The sample itself is normally in the vacuum of the accelerator. Not shown in the diagram is a periscope whose principal element is a mirror placed at 45 degrees to the electron beam and containing a central hole through which the electron beam passes. With this device, visual inspection of the sample during radiation is possible. Also not shown is a magnet with which fields of about 2 kilogauss can be applied to the sample. One pole

beam. Fine leads attached to the sample permit a variety of measurements to be made. Electrical conductivity, Hall coefficients, magnetoresistance, etc. may be measured either during actual radiation or between successive radiations. Thermocouples and a temperature controlling system allow these measurements and the bombardment itself to be carried out in the temperature range between 20° and 500° K. All of this equipment is in an ordinary laboratory space. The problem of radiation shielding is reduced merely to a problem of making sure that the electrons from the accelerator reach the lead radiation shield which reduces the X-ray intensity to a safe for the operating personnel.

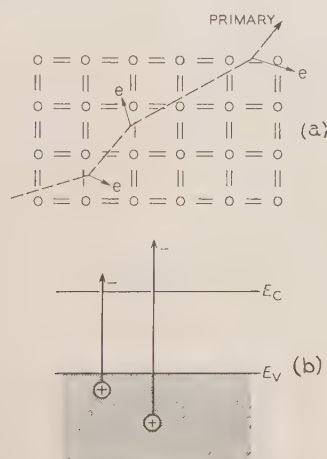


Fig. 2. The ionization process, (a) electron-electron collisions, (b) energy-band representation of the process.

## 2. Electron-Electron Interaction

This machine has been used in two very different ways. They correspond to the two very different interactions which electrons may have with a solid substance. These two interactions will be described separately. One is the process of ionization; the other, the process of atomic displacement. First consider the ionization process. In this case, as shown in fig. 2(a), a high-energy electron enters the crystal and makes collisions fairly frequently with electrons which may be visualized as forming the bonds in this solid lattice. Of course, the collisions may be with electrons more tightly bound to the

atoms than just these valence electrons, but ultimately any higher energy electron which is formed will in a cascade process dissipate a large part of its energy in ionization of the valence bond electrons. From the energy band point of view, this kind of excitation is shown in fig. 2(b). Electrons from a normally full valence band are lifted up into a normally empty conduction band. The valence band states which have lost electrons now contain holes. Two transitions are shown in the illustration: one in which an electron from very near the top of the valence band makes a transition to very near the bottom of the conduction band and one in which an electron from deeper in the valence band is lifted to a higher energy in the conduction band. Any combination of such transitions can occur; but, following the excitation, the electron which is lifted high above the conduction band edge will lose energy very rapidly until it comes to an energy state just above the bottom of the conduction band. The hole will rise to the top of the valence band as the electrons above it in the valence band fall into the energy state the hole originally occupied. In a very short time (in the order of  $10^{-12}$  seconds) the electrons and holes which are produced in the ionization process occur as electrons and holes near the bottom of the conduction band and the top of the valence band respectively. This ionization process can be considered a generation process for holes and electrons. The generation rate is approximately

$$g = \frac{\frac{dE}{dX} i}{\epsilon e}$$

where  $dE/dX$  is the rate of loss of energy of the primary electron beam,  $\epsilon$  is the average energy required to produce a hole-electron pair,  $e$  is the electronic charge, and  $i$  is the current density in the primary electron beam. In germanium  $dE/dX \approx 10$  MeV/cm, and  $\epsilon$  is  $\approx 3$  eV. For beam densities of  $100 \mu\text{A}/\text{cm}^2$  the generation rate is approximately  $2 \times 10^{21}$  pairs/ $\text{cm}^3$  sec.

Following a sudden turn-on of the primary electron beam there is a buildup in the concen-

tration of electrons and holes toward a steady of the magnet has an axial hole for the electron state value. The steady state is the balance between the generation process described above and a recombination process through which the electrons return to the valence band and eliminate holes. The recombination process may be a direct transition of the electrons from conduction to valence band with emission of a quantum of light. It is more likely, however, to be a two step process involving an impurity as an intermediate state. Either of these recombination processes, the direct process or the indirect one via impurities, is of great importance from the standpoint of semiconductor physics. The recombination process can be characterized by a time constant  $\tau$  which typically is between  $10^{-8}$  and  $10^{-4}$  seconds. The steady-state hole and electron concentrations produced by the electron beam are then

$$\Delta n = \Delta p = g\tau.$$

The  $\Delta$ 's indicate that the beam may be generating holes and electrons in addition to concentrations originally present in equilibrium. For  $\tau = 10^{-6}$  sec,  $\Delta n \approx 2 \times 10^{15}/\text{cm}^3$ .

Both the electrons and holes contribute to the electrical conductivity. The densities we have estimated above are small compared with the density of atoms in the solid; however, they may be quite significant compared with the densities of holes and electrons in the material at equilibrium, and will result in significant changes in the electrical conductivity. In metals one would expect to see no conductivity change, both because the equilibrium carrier density is so much higher and because  $\tau$  is expected to be much shorter. In insulators, which differ from semiconductors in the size of the gap between valence and conduction bands, one might expect large changes in conductivity. In good insulators, however, one has to be concerned with buildup of space charge from electrons which stop in the sample—a problem which may make measurement of conductivity difficult and may cause destructive breakdown of the crystal.

## 2.1. TRANSIENT CONDUCTIVITY

One could measure the steady-state change in conductivity under radiation and from  $\Delta n$  and  $\Delta p$  have a measure of  $\tau$ . It is more satisfactory to measure the transient in conductivity which follows the turnoff of the beam—a transient which gives  $\tau$  directly. At Bell Telephone Laboratories, G. K. Wertheim<sup>1)</sup> is using the Van de Graaff in this way to study the details of recombination processes in the semiconductors. Fig. 3(a) shows the circuit used in these measurements. A constant current is supplied the sample from a battery and a suitably large resistance. The voltage across the sample is observed using a cathode follower, chain amplifiers, and a Tekronix 517 oscilloscope.

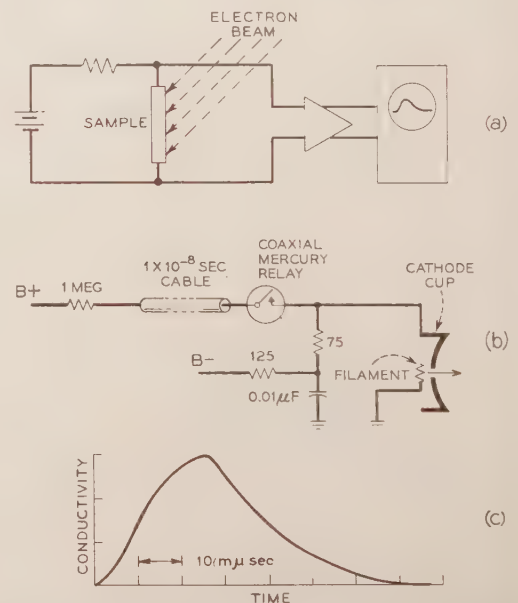


Fig. 3. Use of the ionization process to study recombination transients in semiconductors, (a) sample circuit arrangement, (b) schematic of the 20-millimicrosecond Van de Graaff beam-pulser, (c) response of a sample to the pulse.

To study short lifetimes, one needs rapid turnoff of the electron beam. Fig. 3(b) is a simplified schematic of the additions to the Van de Graaff terminal which have been made for this purpose. The Van de Graaff cathode cup which serves as a grid is biased to cut off the Van de Graaff's beam. This requires a negative

<sup>1)</sup> G. K. Wertheim and W. M. Augustyniak, Rev. Sci. Inst. 27 (1956) 1062.

voltage of only about 75 volts. A delay line, which in this case is a length of cable, is charged to a positive potential of about 95 volts. At the time when one wishes to apply the pulse, a coaxial mercury relay is closed and this length of cable is discharged to a 75-ohm load, raising the cathode cup potential to about the potential of the filament for a period which is twice the length of time required for a pulse to travel down the cable. In this way one can provide a pulse which has a rise time that is limited by the 75-ohm impedance and by the capacity of the cathode cup with respect to terminal ground, about  $40 \mu\mu\text{F}$ . The rise time of the pulse should be about three millimicroseconds. Its duration in Wertheim's case is about 20 millimicroseconds. A photograph of an oscilloscope trace for a sample of rather short lifetime responding to the 20-millimicroseconds pulse is shown in fig. 3(c). Note the buildup of conductance in the sample, tending toward a steady-state value during the time the pulse is on. The beam is cut off before the steady state is reached and one sees the decay of conductance. In this case the decay time is about 12 millimicroseconds. (Electrostatic scanning is not used in pulse measurements since it is essential to have uniform sample coverage in a single pulse. A foil is introduced to scatter the beam, the deflection plates being used only to optimize the scattered beam pattern with respect to the sample.)

Pulses somewhat longer than 20 millimicroseconds can be provided by increasing the length of the delay cable. However, since this cable must all be contained within the high-voltage terminal, pulses longer than about 0.1 microsecond become rather unfeasible. Between the 0.1-microsecond and about 10-microsecond ranges a lumped delay line can be used in place of the delay cable. The repetition rate of the delay-line pulser is controlled by a light flash from a chopper external to the tank of the Van de Graaff. The repetition rate can be varied over a range of one pulse-per-second to about 300 pulses per second, the high limit being set by the operating time of the coaxial mercury relay.

### 3. Electron-Nuclear Interaction

Now let us consider the interaction of high-energy electrons with the nuclei of the semiconductor rather than with its electrons (see fig. 4). The high-energy electron passing through the crystal may make a collision with a nucleus.

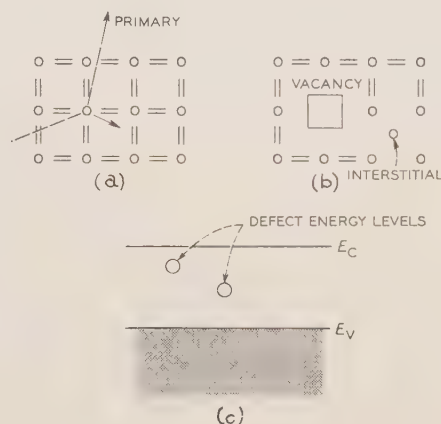


Fig. 4. The process of atomic displacement, (a) electron-nuclear collision, (b) vacancy-interstitial formation, (c) vacancy-interstitial energy levels in the band representation.

This is not a nuclear reaction in the sense that the electron actually enters the nucleus, but the electron can come very close to the nucleus and make a very hard impact with it. The electron is, of course, scattered as in fig. 4(a) and the nucleus is given an impulse. Because of the large difference in their masses the energy transferred is a very small part of the electron energy. It cannot exceed 60 eV for a germanium atom struck by a 1-MeV electron. Under favorable conditions, the energy will be sufficient for the atom to be moved from its location in the crystal and placed in an interstitial position as shown in fig. 4(b). A vacancy is formed in the place from which the atom was removed. The diagram does not show the way in which the electron bonds around the vacancy and interstitial exist in this case. These are features of the problem which we do not yet understand. From the standpoint of the energy-band diagram, these defects show up as isolated energy levels in the forbidden energy gap between the valence and conduction bands in

the semiconductor as illustrated in fig. 4(c). In this location they have the capacity to change the concentration of electrons and holes in the material at equilibrium and to act as recombination centers for changing the lifetime by the process previously described.

One of the principal advantages of electron bombardment for the introduction of defects in solids is the inherent simplicity of the damage

mental point of view in semiconductors and in other materials as well. A 1-MeV electron should be capable of producing these defects in almost all solid substances except for those of highest atomic mass. In many organic substances the permanent effects of ionization (of which there are none in germanium and silicon) will greatly overshadow the effects of atomic displacement by direct collisions with nuclei. Among semi-

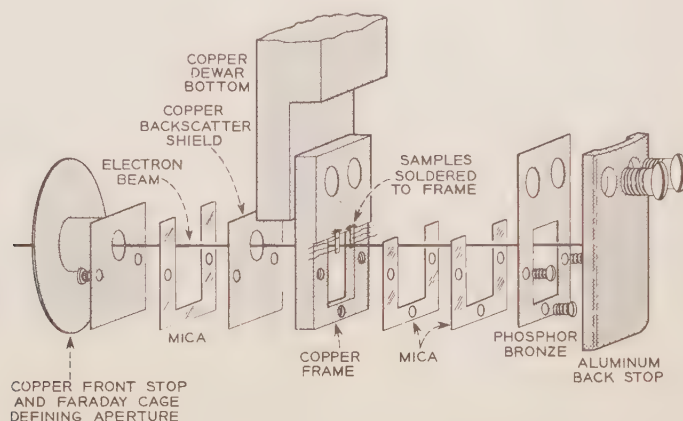


Fig. 5. Faraday-cage type sample holder for measurement of electron flux.

that results. This arises from the fact that the energy imparted to a nucleus in a collision is so small, at most only a few times the 10 to 20 volts minimum energy required to form a vacancy-interstitial pair, that it is very rare to find a collision producing multiple defects. This is in contrast to the situation in heavy particle bombardment where many kilovolts of energy may be transferred in a single collision in which the primary struck atom plows on through the solid, displaces more atoms and produces a more complicated system of multiple defects. The presence of vacancies and interstitials in a semiconductor is detectable in a great many ways. One which yields the most detailed atomic picture of the defects is microwave spin-resonance. The simplicity of the results of such experiments in silicon<sup>2)</sup> is confirmation of the simplicity of the defects introduced by million-volt electron bombardment.

Vacancies and interstitials are special kinds of defects which are of interest from a funda-

conductors and metals, the cross section for defect formation will vary with electron energy, with atomic mass and with details of crystalline binding; but is typical of the order of a few barns at 1 MeV.

One important experimental problem is that of determining the integrated flux of electrons which a sample has received. Since we often are concerned with working at low electron energies where the efficiency of creation of defects becomes very small, it is important to supply electrons at a high rate to produce a significant effect in a reasonable time. The sample must consequently be very small (in the order of half a millimeter by two millimeters) to allow it to be kept cool in the presence of the large amount of energy being dissipated in it through electron-electron interaction. Small size aggravates the problem of flux measurement. Fig. 5 shows the way which we have devised for determining the primary electron flux. To the left in this diagram is a copper front-stop and Faraday cage defining aperture. The beam is electrostatically scanned

<sup>2)</sup> G. Bemski, Bull. Am. Phys. Soc. 3 (1958) 135.

to cover a reasonably large area on this front-stop and a part of that scanned beam passes through the defining aperture which is typically about 2.5 millimeters in diameter. It then passes through a copper back-scatter shield whose aperture is larger than the aperture in the front-stop and reaches the samples which are shown here soldered to a frame connected to the bottom of the copper dewar that was shown in fig. 1. The beam passing through the samples is caught on an aluminium backstop. The copper back-scatter shield, the sample frame, the samples, and the aluminium backstop are all connected together electrically as a Faraday cage. The copper frontstop is electrically insulated from these other pieces and has a wire

in density by simply dividing its current by the area of the hole through which this current has passed. This method has been found adequate for getting reasonably reliable measurements of beam density and the resulting integrated flux at the sample.

### 3.1. ELECTRICAL PROPERTY CHANGES

The equilibrium electrical conductivity is quite sensitive to the radiation produced defects in semiconductors. The conductivity is

$$\sigma = ne\mu_n + pe\mu_p$$

where  $n$  and  $p$  are the equilibrium densities of electrons and holes (the measurements are to be made with the electron beam turned off),  $\mu_n$  and  $\mu_p$  are the mobilities of the carriers, and  $e$  is the electronic charge. Both the carrier density and the mobility change under electron bombardment. This is shown in fig. 6 for an  $n$ -type sample which initially contains about  $3 \times 10^{14}$  electrons/cm<sup>3</sup> at equilibrium and a negligible number of holes. The ordinate is the ratio of the conductivity to the original conductivity, and the carrier concentration to the original carrier concentration as determined from Hall effect measurements. These quantities are plotted against the integrated electron flux. Both ratios start by changing linearly. One electron is removed from the conduction band for each defect energy level introduced by the bombardment. This is reflected in the decrease in the carrier concentration. The loss of conduction electrons contributes, through the relationship above, to a decrease in the conductivity, which is further decreased by a decrease in mobility of the remaining electrons due to the bombardment defects. As a result, the conductivity changes more rapidly on a percentage basis than the carrier concentration. After a time the linearity in both the curves begins to fail due to (among other things) nonuniformity in the distribution of the defects through the thickness of the sample. The slope of either of these curves we may use as a means of measuring the rate at which defects are produced in the sample. A rapidly changing carrier concentration

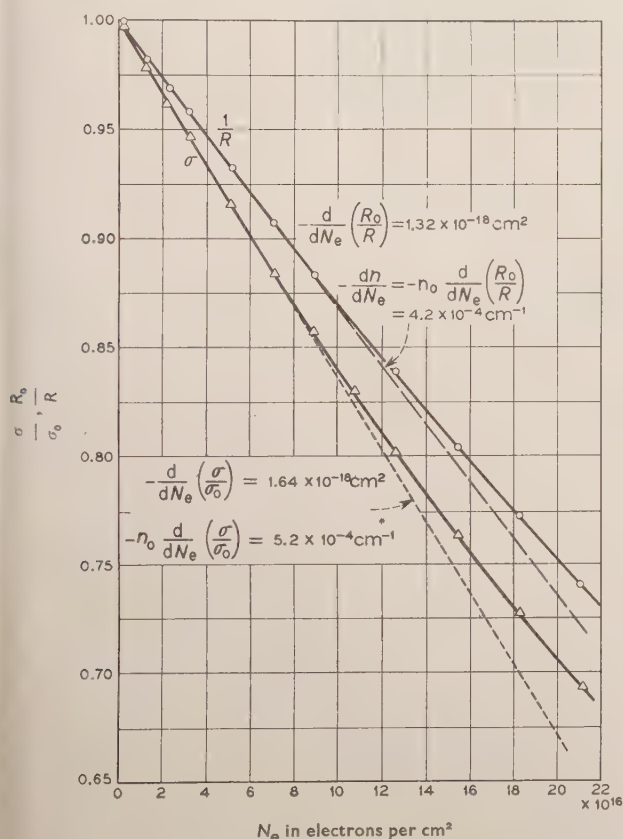


Fig. 6. Carrier concentration and conductivity changes resulting from electron bombardment.

attached for carrying off the charge it stops. Any electron beam which is collected on the pieces beyond the defining aperture is calibrated

or conductivity means vacancies and interstitials are being added rapidly to the crystal.

### 3.2. RECIPROCITY

One of the interesting questions to be settled is that of reciprocity. In many chemical systems it has been found that the effects of radiation depend in a very important way on the rate at which a radiation dose is delivered. There is a

those shown at the top end of this slide, has found very marked failures in reciprocity in his samples in experiments done in very much the same way. This aspect of the problem is a very interesting one to follow up more completely.

### 3.3. TRESHOLD FOR DEFECT FORMATION

Another facet of the damage process we have investigated has been in connection with the

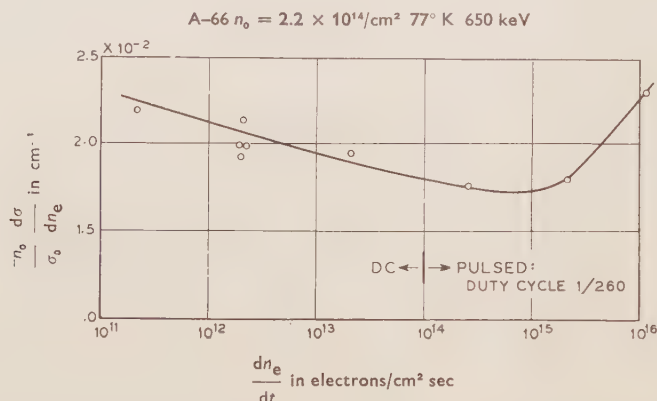


Fig. 7. Variation of the defect formation probability with rate of introduction of electron flux.

rate effect or a failure of reciprocity. In fig. 7 the results of such an experiment are shown. The rate of change of the carrier concentration with electron flux is plotted as the ordinate. This is just the slope of a curve such as that shown in fig. 6. It is plotted against the rate at which the electrons are introduced to the sample. This is essentially the current density which was varied over five orders of magnitude with a maximum of about 1 milliamper/cm<sup>2</sup>. It is apparent on this linear versus logarithmic plot that the effectiveness with which vacancies and interstitials are introduced into the crystal changes only very little with the dose rate over this range of current density. There is an indication at the high current end of this diagram that the effect is beginning to increase. In view of the scatter one sees in the group of points at about  $2 \times 10^{+12}$  electrons/cm<sup>2</sup>/sec, one is somewhat skeptical about attaching too much significance to this rise. However, MacKay at Purdue University<sup>3</sup>), working at current densities as much as a factor of a hundred higher than

threshold for defect formation<sup>4</sup>). It seems obvious that there is some electron energy below which an incident electron is incapable of imparting enough energy to an atom of the crystal to move it to an interstitial position. This energy we denote as the threshold energy. The results of a threshold experiment are shown in fig. 8. Here again the ordinate is the rate at which defects are introduced. In this case the rate of change of the conductivity is used as a measure of the defect formation rate. This is plotted against the primary electron energy. There are four samples shown here, two of them 63 microns (two and a half mils) thick and two of them about 15 microns thick. In each case these have been irradiated in pairs. For one of the two thicker samples, the electron beam arrived at the surface in a (111) crystallographic direction; in the other sample, in the (110)

<sup>3</sup>) J. Kortright and J. MacKay, Bull. Am. Phys. Soc. 3 (1958) 142; and J. MacKay, private communication.

<sup>4</sup>) W. L. Brown and W. M. Augustyniak, Bull. Am. Phys. Soc. 2 (1957) 156.

direction. It is clear that there is a significant difference between them. The rate of defect formation is higher in that sample where the electrons arrive in the (111) orientation than it is for the (110) sample. This is just the result to be expected from a very simple examination of the crystal structure which is involved. Surprisingly there is no real difference discernible in the energy at which these defects begin to be

principally in the front part of the samples. The electrons are degraded in energy sufficiently that by the time they reach the back they are incapable of producing defects. Again for the thin samples the (111) orientation damages more readily in comparison with the (100). Again the threshold energy shows no significant difference. These features of the threshold and orientation dependence of the defects formation

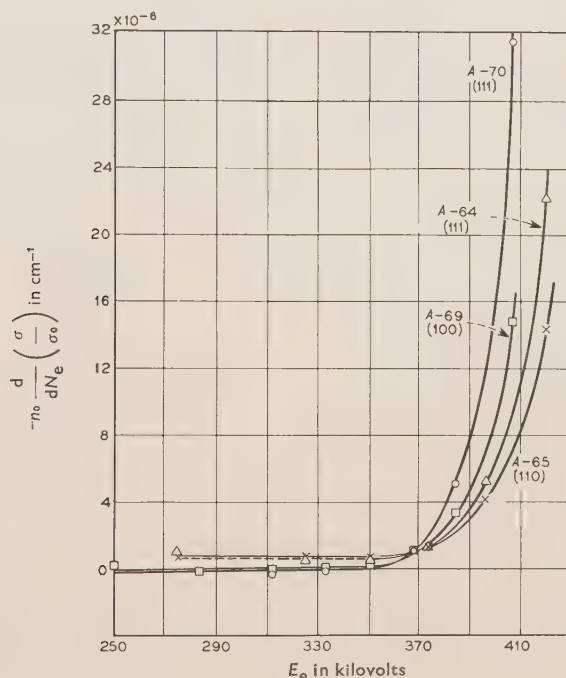


Fig. 8. The threshold and orientation dependence of the defect formation probability.

produced. The threshold energy does not seem to depend on orientation as does the damage rate at energies just above. The threshold energy is about 360 kilovolts in this measurement which was carried out at liquid nitrogen temperature. One of the possibilities for this failure to observe an expected difference in threshold energy was that multiple scattering of the electrons played a very large role. The thinner samples should be more significant in this respect. The average damage rate is greater in the thinner samples because at energies very close to threshold, defects are being introduced

in germanium are still poorly understood.

There are other features of this work which one might describe such as the temperature dependence of the rate of damaging; the energy dependence over a much wider range than has been discussed; the annealing kinetics of the defects which have been introduced; and so on. However, the few cases we have examined will give an indication of the way in which a small electron Van de Graaff is being used to explore the formation of defects in semiconductors and to examine the transients in recombination processes in semiconductors.

## A TRANSISTOR CIRCUIT FOR FAST COINCIDENCE MEASUREMENTS

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A coincidence circuit is described which has a short resolving time ( $< 5 \mu\text{sec}$ ), and a high rejection of non-coincident pulses; coincident one volt pulses trigger the circuit while single 40 V pulses do not.

Use is made of germanium diodes and transistors, and pulse shaping is accomplished by means of small inductances. The resulting circuit is sensitive, compact, and consumes little power.

## 1. Introduction

In Nuclear Physics experiments it is often necessary to detect light signals in a scintillator which coincide with signals from other scintillators. Normally measurement of coincidence<sup>1)</sup> involves forming the output from photomultipliers into narrow pulses of constant amplitude and mixing them so that an output is obtained only when pulses overlap.

The present circuit uses this basic technique but employs diodes and transformers to shape the pulses and a transistor amplifier to provide the output signal. The circuit will respond to input pulses of one volt or greater and has a resolving time of  $3 \mu\text{secs}$ .

Fig. 1 shows the circuit (a) together with typical waveforms (b-f). The input is connected to the photomultiplier anode by cable matched at the sending end. Signals consisting of negative voltage pulses, shown at (b), pass through the condenser C and cut-off diode  $D_1$ , thus removing a current of 10 mA from the transformer primary winding. The voltage generated across the transformer comprises a negative pulse of fixed amplitude and width (1 volt and  $4 \mu\text{secs}$ ) with a critically damped overswing, as shown at (d).

The transformer secondary is connected to a conventional diode coincidence circuit so that only when pulses appear simultaneously on

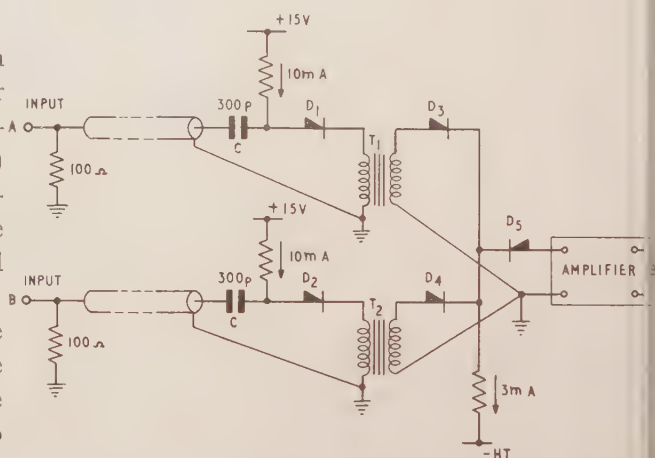


Fig. 1. (a) The circuit.  $D_1$  and  $D_2$ : 0A90.  $D_3$ ,  $D_4$  and  $D_5$ : CV2290.

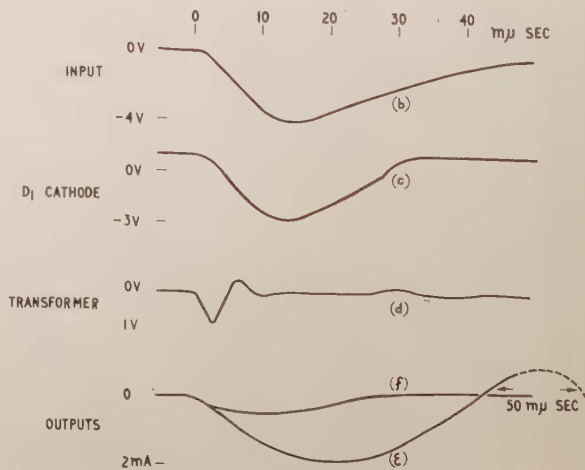


Fig. 1. (b-f) Waveforms.

<sup>1)</sup> I. A. D. Lewis and F. H. Wells, *Millimicrosecond Pulse Techniques* (Pergamon Press, 1954).

both transformers do negative signals appear at the input of the amplifier. Since these signals have short duration, less than  $4\text{ m}\mu\text{sec}$ , the amplifier is arranged to stretch them giving an output which lasts for  $50\text{ m}\mu\text{sec}$ . Curve (e) in fig. 1 is a typical output pulse for coincident inputs, while curve (f) shows the break-through for a single 20 volt input.

## 2. Circuit Design

### 2.1. DIODES

Many of the difficulties arising in the design of fast coincidence circuits are due to the inadequacy of diodes for switching fast pulses.

This is demonstrated in fig. 2 where the voltage across a diode is recorded when its current is switched on and then off.

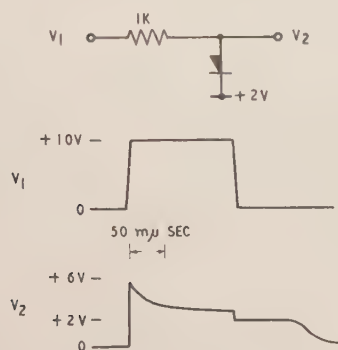


Fig. 2. Switching transients in diode type CV448.

The initial peak in the voltage shows that for very narrow pulses the diode forward-resistance is much greater than would be expected from D.C. measurements. Furthermore, when the input is removed the diode conducts a reverse current until a charge, stored during conduction, either decays or is extracted. Thus for rapid operation a forward transient voltage is required to initiate conduction and a reverse transient current is required to terminate it.

Some typical values of diode transients together with the charge stored and the maximum allowable reverse voltage are listed in table 1. It can be seen that the diode which has the best transient properties (CV 2290) suffers from limited reverse voltage rating, and there does not appear to be any commercially avail-

able diode which combines the advantages of both low transients and high rating.

Referring again to fig. 1(a), diodes  $D_1$  and  $D_2$  must withstand a reverse voltage equal to the maximum input pulse amplitude. Somewhat inferior transient properties can be tolerated in them, however, because the charge extracted during switch off is merely equivalent to an increase in the 10 mA gate current, while the forward voltage transient slightly lengthens the recovery time of the circuit. Diodes type 0A90 are used, and although the charge stored in them is large,  $100\text{ m}\mu\text{coulombs}$ , the rate at which it can be extracted is limited, probably by the way in which this charge is distributed in the diode, so that even with large reverse voltages less than  $25\text{ m}\mu\text{coulombs}$  can be removed in the first  $5\text{ m}\mu\text{sec}$ . Therefore carrier storage in the diode contributes less than 5 mA to the current change in the transformer, and this is virtually unaffected by input pulse amplitude. Thus the voltage pulse appearing across the transformer primary winding is sensibly independent of input pulse amplitude.

Diodes  $D_3$  and  $D_4$  constitute a coincidence gate and must have good transient properties. The reverse voltage rating of these diodes, however, need be no greater than the pulse amplitude on the transformer, which is about 1 volt. Thus diodes Type CV 2290 can be used.

TABLE 1  
Response of diodes to 10 mA pulses

| Diode   | Peak volts at switch on (volts) | Charge stored $\mu\text{coulombs}$ | Max. current at switch off (mA) | Rated reverse voltage (volts) |
|---------|---------------------------------|------------------------------------|---------------------------------|-------------------------------|
| CV 448  | 4                               | 200                                | 7                               | 100                           |
| 0A 90   | 1.3                             | 100                                | 5                               | 30                            |
| CV 2290 | 0.6                             | < 10                               | < 1                             | 3                             |

### 2.2. TRANSFORMER

The main function of the transformer is to generate narrow pulses of standard shape. A short-circuited delay-line is suitable but the transformer is preferred because of its compactness, and because it allows the current flowing into the amplifier to be separated from the

large currents flowing in the transformer primary, thus avoiding the unwanted coupling that may occur in a common earth return.

The transformer comprises two 25 s.w.g. wires each threaded once through a ferroxcube tube of length 3 cm, outer diameter 5.6 mm and inner diameter 1.5 mm. Ferroxcube grade B5† is used giving an inductance of  $0.1 \mu\text{H}$  for each winding and this resonates near 200 Mc/s when connected in circuit. The circuit losses are sufficient to damp the coil, i.e. about  $80 \Omega$ .

### 3. Effects of Spurious Inputs on Coincidence Measurements

The circuit is designed to detect coincident events in the presence of a large number of spurious signals, which are liable to cause errors in a number of ways as follows:

A false output occurs when spurious signals appear simultaneously at both inputs. However the probability of this occurring is easily determined<sup>1)</sup> and a correction can be applied to the measurements.

Less determinate errors would occur if large inputs were able to break through the diodes and affect the amplifier, but this is unlikely in the present circuit. For example a 20 volt input at A with no signal at B produces a 1.2 volt pulse on transformer  $T_1$ . Diode  $D_3$  is cut off and its standing current of about 1.5 mA now flows through  $D_4$  giving 50 mV across the diode and 150 mV across transformer  $T_2$  making a total of 0.2 V which is insufficient to bring  $D_5$  to full conduction. The signal applied to the amplifier in such a case is therefore small.

True coincidences may be missed if they are preceded immediately by a background signal at either input. This error is reduced by making the capacitance of C small to give the circuit a short recovery time. Thus a typical 4 volt input at A cuts off  $D_1$  and the capacitance re-charges rapidly, as in fig. 1(c), returning the diode to conduction in less than  $40 \mu\text{sec}$ . The circuit is then ready for another input.

† Mullard FX 1487.

### 4. Amplifier Design

Signals applied to the amplifier, shown in fig. 3(a), consist of 3 mA pulses of  $4 \mu\text{sec}$  duration which are stretched by the amplifier

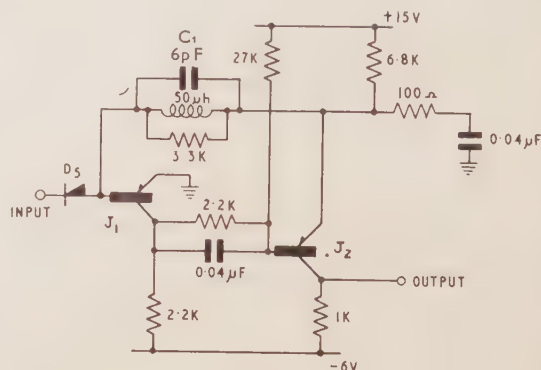


Fig. (3a)

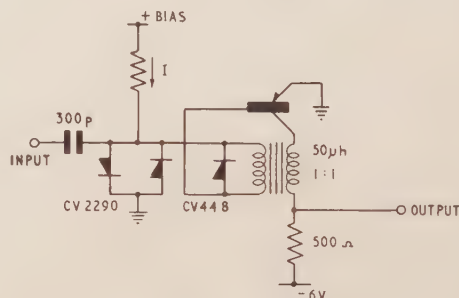


Fig. (3b)

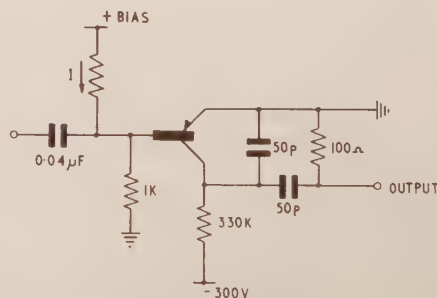


Fig. (3c)

Fig. 3. (a) Amplifiers. Transistors SB346. (b) Blocking oscillator. Transistor 2N393. Transformer comprises 7 + 7 turns on ferroxcube pot-core Type FX1011 (Mullard). (c) An avalanche trigger circuit. Selected 0C44 transistor.

to 2 mA pulses lasting for  $50 \mu\text{secs}$ . The amplifier gain is defined by feedback from the emitter of  $J_2$  to the base of  $J_1$ . Initially the signal is integrated by the capacitance  $C_1$  but after about  $20 \mu\text{sec}$  the shunting effect of the

inductance becomes appreciable and the output returns to zero after an overswing, the 3.3 k $\Omega$  resistor damping the resonant circuit. The inductance in the feedback circuit defines the integration time of the amplifier as  $\pi\sqrt{LC} \approx 50$   $\mu$ sec. A capacitance-resistance feedback circuit would be simpler but has the disadvantage of an output which falls exponentially with time, so there is a danger of the circuit becoming saturated by the integration of a fast sequence of inputs.

Diode  $D_5$  in series with the input prevents the positive overswing of the transformers affecting the amplifier, and gives discrimination against small spurious inputs.

A typical amplifier output is shown in fig. 1(e) while fig. 1(f) shows a spurious output derived from a large pulse at only one input. Normally some additional amplification is required with means for rejecting low-level spurious signals. This is provided by the blocking oscillator trigger circuit in fig. 3(b), whose triggering level is set by the bias current,  $I$ , at a value greater than the amplitude of the spurious signals. This circuit provides a 5 volt output pulse with a rise time of 100  $\mu$ sec and less than 1  $\mu$ sec dead time. An alternative trigger circuit is the transistor avalanche circuit<sup>2)</sup> of fig. 3(c), which produces 20 volts in less than 10  $\mu$ sec but takes 3  $\mu$ sec to recover.

## 5. Performance of the Circuit

The circuit has been tested with pulses derived both from a pulse generator and a scintillation counter. 0.8 volt pulses are sufficient to operate the circuit but with these small pulses the performance is affected by the rise time. For satisfactory operation therefore inputs should be greater than 2 V so that the diodes are switched off by the initial fast-rising part of the pulse.

Fig. 4(a) shows how the counting efficiency is affected when two input pulses are separated by short delays, using a pulse generator providing 5 volt signals with 7  $\mu$ sec rise times. The

delay curves in fig. 4(b) was taken with E.M.I. 6097B photomultipliers with scintillators set up in a 150 MeV Proton beam from the Harwell cyclotron. The relevant signals were greater than 4 V in amplitude, rising in about 12  $\mu$ sec. The sides of the delay curve have deteriorated,

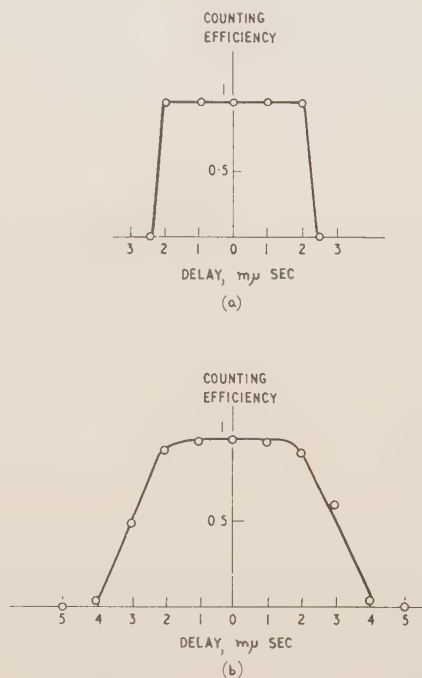


Fig. 4. Delay Curves (1 mA bias current). (a) For standard 4 volt pulses. (b) For photomultiplier pulses.

due to a time jitter of 2  $m\mu$ sec in the photomultipliers. This is confirmed by a close examination of the photomultiplier outputs when operated from a pulsed light source.

Fig. 5 shows the variation in resolving time and counting rate with the blocking oscillator bias current,  $I$ . Increasing bias current lowers the circuit sensitivity, and the resolving time falls because the circuit triggers from the narrow peaks of pulses generated in the transformers. Bias currents greater than 1.5 mA are not acceptable because the counting efficiency then falls appreciably below unity. On the other hand using very small bias currents the circuit becomes unduly high sensitive so that it triggers from small voltage ripples at the input, and the apparent counting efficiency rises above unity. The bias is normally set to a central value on the

<sup>2)</sup> G. B. B. Chaplin *et al.*, A method of Designing Avalanche Transistor Trigger Circuits, Proc. I.E.E. Part B (1959).

plateau of the curves where the performance of the circuit is almost independent of change of components, voltage supplies, and temperature.

## 6. Development of the Circuit

A two-input circuit has been discussed, but the circuit can be extended to take more inputs with little deterioration in performance. For example, 4 inputs have been used successfully.

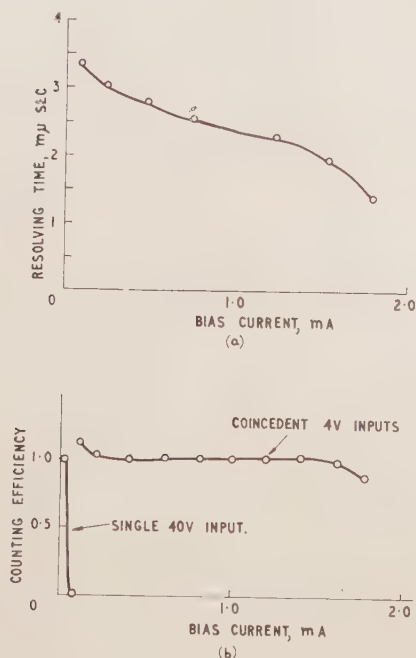


Fig. 5. Effect of blocking oscillator bias current using standard 4V input pulses.

Fig. 6 shows how the circuit is modified to include a veto input. The transformer,  $T_1$ , is constructed with two turns on both its primary and secondary. This provides  $8\text{ }\mu\text{sec}$  pulses timed to overlap the  $4\text{ }\mu\text{sec}$  pulses generated in the single turn windings of transformer  $T_2$ . Also shown in fig. 6 is a remote switching system which enables diodes in the coincidence circuit to be permanently cut-off. This facility is

useful for aligning the delays in the various inputs, and it enables the circuit to be used for counting the spurious pulses on any one input.

## 7. Conclusions

The circuit is compact, consumes little power, and has proved very reliable in use. The sensitivity is superior to most thermionic valve circuits and in many instances allows distributed amplifiers to be dispensed with.

The use of an inductance, instead of a delay line, for shaping the pulses results in a saving of space and avoids errors caused by the large currents flowing in the earth wires.

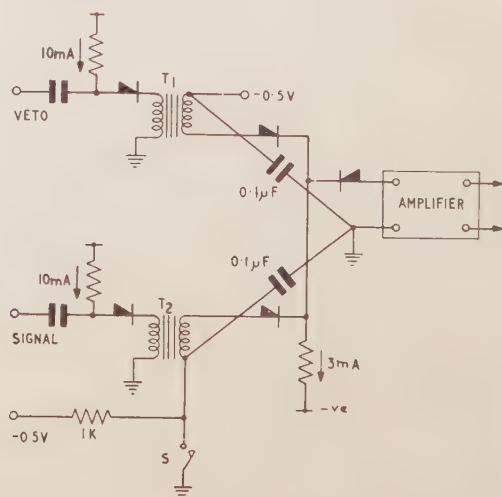


Fig. 6. Part of a coincidence circuit with a veto input and manual switching.

## Acknowledgements

The authors acknowledge the interest and valuable assistance of Dr. T. Gooding and Mr. D. Jones in both the construction and testing of the circuit.

They are also indebted to Mr. A. R. Owens and Mr. E. H. Cooke-Yarborough for several useful suggestions.

## A HIGH ENERGY GAMMA RAY SPECTROMETER†

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A gamma-ray spectrometer for use in the energy range 50 to 150 MeV is described. The spectrometer employs a rather conventional lead glass Čerenkov counter in coincidence with a scintillation-counter telescope as a detector. The circuits which detect and analyse the gamma rays are

of simple and reliable design, and use only crystal diodes and switching transistors as their active elements. The results of a calibration experiment using an electron beam are presented to demonstrate the performance of the apparatus.

### 1. Introduction

The general theory and operation of total absorption gamma-ray spectrometers have been discussed in considerable detail by Mott and Sutton<sup>1</sup>). Several principles mentioned by them have been incorporated in the design of the present instrument. A cylinder of lead glass viewed by a single photo tube converts the energy of the gamma ray into a current pulse. The height of the output pulse of this counter is then used as a measure of the energy of the incident gamma ray. The entry of a gamma ray into the lead glass counter is detected by a scintillation-counter telescope consisting of an anticoincidence counter, a lead converter, and two coincidence counters. The counter telescope opens an electronic gate to permit time coincident pulses in the glass counter to be transmitted to a pulse-height analyser. The electronic circuits used to process the information pertaining to the gamma rays are rather specialized; however, by using transistors and diodes which have only recently become available, extreme

simplicity and reliability have been achieved without sacrificing any speed of operation. In fact, many of the circuits perform their functions in times which would be very difficult to realize using conventional vacuum tube techniques. The resolution of the spectrometer described below is rather poor, but this results from the properties of the lead glass detector rather than from the associated circuitry. In fact, these circuits could be applied to a wide variety of detectors including those with much longer characteristic times than that described here.

### 2. The Spectrometer

In fig. 1 is shown a drawing of the detection apparatus. The counter telescope and lead converter are mounted as near the lead glass as possible to minimize the effects of multiple scattering on the electron-positron pair created in the converter by an incident gamma ray. The thickness of the 3 mm lead converter is determined by a compromise between the need for a large detection efficiency and an optimum pulse-height resolution. Although in this case the size of the piece of lead glass was dictated by that which happened to be available in the laboratory, several guiding principles were used

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<sup>1</sup>) W. E. Mott and R. B. Sutton, *Handbuch der Physik*, Vol. 45 (Springer Verlag, 1958) p. 164.

in the design of the Čerenkov counter itself. Because our object was to measure the gamma-ray spectra produced in the charge exchange scattering of negative pions<sup>2)</sup>, and because there are no prominent features to be resolved in these spectra, poor resolution was not considered a major handicap. Instead a relatively small piece of glass which would contain approximately 70% of a 185-MeV cascade shower

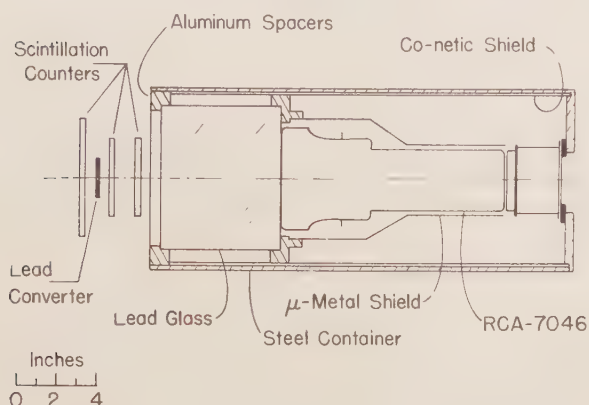


Fig. 1. Section view of the gamma ray spectrometer.

generated in the converter<sup>3)</sup> was considered satisfactory. This small piece of glass could be viewed by a single photo tube with fair light collection efficiency, and permitted complete magnetic shielding of the photo tube without invoking too cumbersome a mounting arrangement. This last requirement was of especial importance because the charge exchange scattering experiments required moving the counter from point to point in the stray magnetic field of the cyclotron.

The determination of the efficiency of the converter as a function of energy has been described in detail elsewhere<sup>2)</sup> and is justified by the very satisfactory results obtained in the charge exchange scattering experiments. It is important that the resolution be constant at all points on the converter, and although a prediction based on the work of Kantz and Hofstadter<sup>3)</sup> indicated that this would be the case, a

subsidiary set of measurements was carried out during the calibration experiment to verify this fact.

The counter itself, as shown in fig. 1, is very compact and relatively easy to handle. Light collection is facilitated by polishing all surfaces of the lead glass cylinder except that viewed by the photomultiplier tube. The cylinder is wrapped in highly polished aluminum to aid in light collection and to give a light tight cover. Optical coupling between the lead glass and photo-cathode is achieved with silicone high-vacuum grease. The seal is maintained by exerting mechanical pressure on the photo tube through its mu-metal magnetic shield. A rubber pad, glued to the inside of this shield, transmits the pressure to the tube. The back cover plate of the magnetic shield permits the high voltage and signal cables, as well as the potentiometers used to adjust the focusing grids of the photo-tube, to be readily accessible when the counter is assembled and in operation.

Considerable care was used to adjust the potential of the focusing grids of the RCA 7046 phototube to give the optimum pulse-height resolution. The final adjustments were made using a monoenergetic electron beam incident upon the counter. The potentials of the focusing grids were adjusted to give a minimum width to the pulse-height distribution produced in this way.

### 3. The Electronic Circuitry

Fig. 2 is a block diagram showing the arrangement of the electronics. Inputs 1 and 2 are for counters which monitor the incident beam intensity for the charge exchange scattering experiments; input 3 is for the anticoincidence counter in front of the lead converter; and inputs 4 and 5 are for the pair of counters between the lead converter and the glass counter. Output pulses to drive scalars are provided to count the double, quadruple, and quintuple coincidences, as well as the quadruple-quintuple anticoincidence. The latter output opens a 20 mμsec gate to allow the pulse from the Čerenkov counter to be analysed in a fifty

<sup>2)</sup> E. L. Garwin, W. Kernan, C. O. Kim and C. M. York, submitted for publication in the *Physical Review*, 1959.

<sup>3)</sup> A. Kantz and R. Hofstadter, *Nucleonics* 12 (1954) 3.

channel pulse-height analyser. The internal gate of the analyser is triggered from the quadruple-quintuple anticoincidence scaler pulse, so that only pulses satisfying the logical requirements are registered, i.e., any very large pulses feeding through the closed diode gate are not analysed.

times on the order of 30  $\mu\text{sec}$ , which results from using unclipped input pulses. Careful studies of counting losses as a function of counting rate were carried out. No losses of greater than 4% were detected at instantaneous rates up to  $6 \times 10^5$  counts per second.

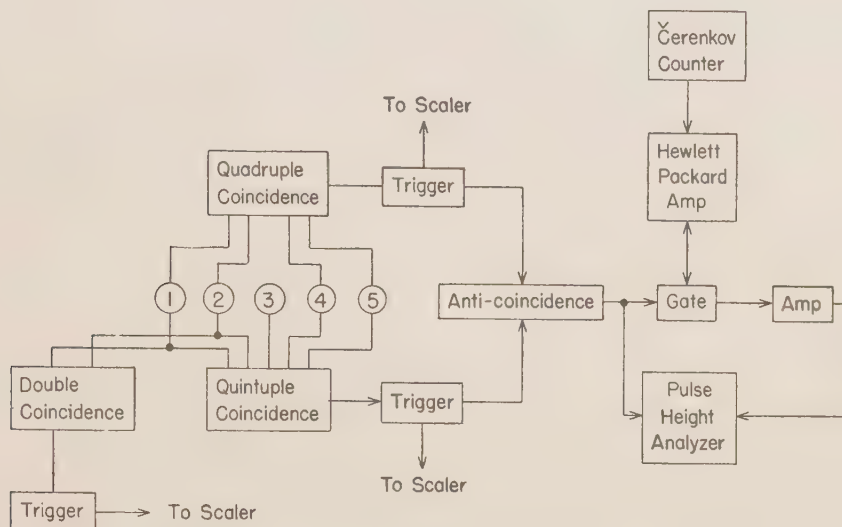


Fig. 2. Block diagram of the electronic circuits used in the spectrometer.

The coincidence circuit inputs are fed directly from RCA 6810 photomultiplier tubes without further amplification, while the signal to be pulse-height analysed is obtained from an RCA 7046 photomultiplier, driving a Hewlett-Packard 460A distributed amplifier. The 6810 tubes are operated at 2500 V, and the 7046 requires 3000 V.

The coincidence requirements are achieved by using standard diode coincidence circuits<sup>4)</sup> employing Transiton S-570-G diodes with 2  $\mu\text{sec}$  recovery time. The output pulses produced by the coincidence circuits are fed into transistorized trigger circuits. The anti-coincidence requirement on the counter preceding the lead converter is obtained by forming a quadruple coincidence not involving that counter, and a quintuple coincidence which does involve it. The outputs of these two circuits are then placed in anti-coincidence. This arrangement has a measured anti-coincidence inefficiency of 1 in 5000. The coincidence circuits have resolving

times on the order of 30  $\mu\text{sec}$ , which results from using unclipped input pulses. Careful studies of counting losses as a function of counting rate were carried out. No losses of greater than 4% were detected at instantaneous rates up to  $6 \times 10^5$  counts per second.

Fig. 3a shows the connection of the diodes required to obtain the various coincidences described above. The 10  $k\Omega$  resistors at the input connectors serve as DC returns for the voltage limiting diodes whose anodes are connected to a —1.3 volt mercury battery. These limiters are necessary to prevent excessive back voltages from appearing across the coincidence diodes, which have a peak inverse voltage of 8 V. The operation of this type of diode circuit is well-known, and will not be discussed further<sup>4)</sup>.

Fig. 3b is the schematic wiring diagram of the trigger circuit employed with each coincidence circuit. The emitter follower, T-1, is introduced to minimize the loading on the output of the coincidence circuit. Transistor T-2 serves as a shunt trigger for the univibrator composed of T-3 and T-4. T-5 is an emitter follower to drive the one-volt negative output, and T-6 is an inverter which delivers positive pulses of seven

<sup>4)</sup> S. de Benedetti and R. W. Findley, *Handbuch der Physik*, Vol. 45 (Springer Verlag, 1958) p. 236.



volts to the Hewlett-Packard fast scalers. The trigger circuit has a well-defined dead time of 100  $\mu\text{sec}$ , and a pulse length of 50  $\mu\text{sec}$ . Both the length and height of the output pulse produced by this trigger circuit are independent of the size of the triggering pulse and of the bias level of the shunt trigger, T-2.

Fig. 3b also shows the arrangement used to detect an anti-coincidence. Transistor T-7 acts as a short circuit when a negative pulse is present at its base. Therefore a quadruple output pulse passes through this gate only in the absence of a pulse in the quintuple circuit.

T-4 acts as a stretching device to lengthen the input pulse. Its anode is connected to a potentiometer which supplies an adjustable back bias, thus providing a means for minimizing pedestal feed-through. The capacitive load presented to the gate is about 30  $\mu\text{F}$ . This value is small enough so that nonlinearities of less than 2% are imposed upon input pulses of up to 7 volts. Transistor T-4 serves as an emitter follower to drive the 2000 ohm input impedance of a linear amplifier with a gain of about ten. This amplifier is required to present the pulse-height analyser with a 50 volt maximum input signal.

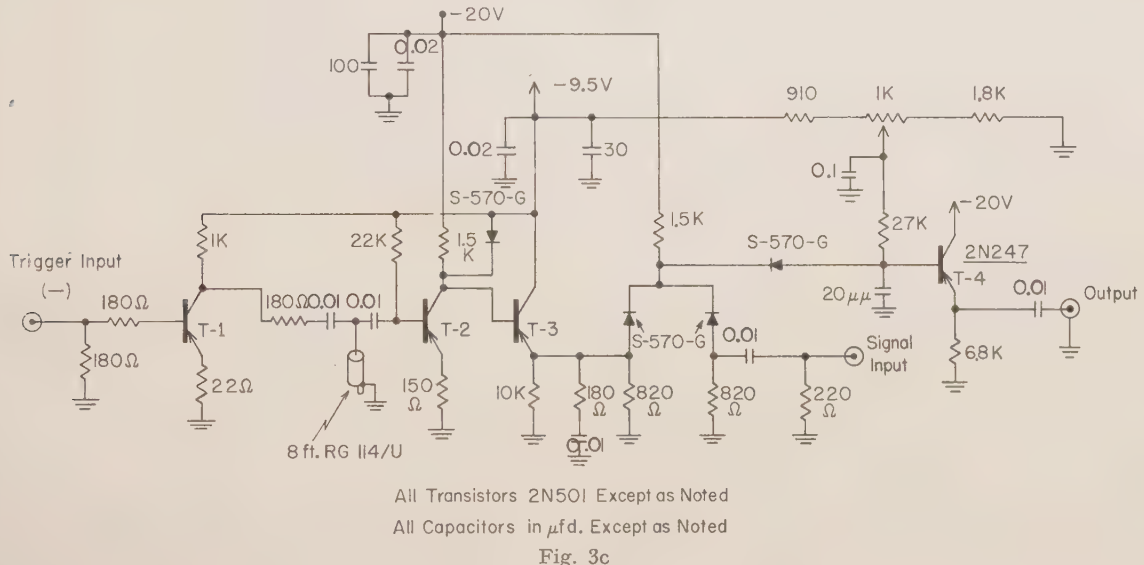


Fig. 3c

(Fig. 3a,b,c). Detailed circuit diagram of the electronics.

Fig. 3c is the circuit of the linear gate. The trigger input is derived from the negative output of the anti-coincidence circuit of fig. 3b. Transistor T-1 in fig. 3c is an inverter, whose output pulse is clipped by a 180 ohm delay line. The length of this delay line sets the open time of the gate circuit, which can be varied from 20 to  $\approx 100 \mu\text{sec}$ . The transistor T-2 squares and limits the pulse applied through the emitter follower, T-3, to one side of the "AND" gate. The gating pulse is 20  $\mu\text{sec}$  long at its top, 30  $\mu\text{sec}$  at its base, and is trapezoidal in shape. The signal pulse to be analysed is applied to the other side of the gate. The diode connecting the junction of the two gate diodes to the base of

#### 4. The Calibration Experiment

A low energy negative pion beam which was known to be very highly contaminated with electrons was used to calibrate the spectrometer. The beam was focused on the counter array with the aid of a wedge magnet. At a given current setting of this magnet a range curve was taken to determine the energy (and hence momentum) of the pions in the beam. The magnet current was then decreased to obtain lower magnetic fields and hence lower momentum particles. Pulse-height spectra in the glass counter were taken at eight such magnet settings to give incident electrons of eight different energies. The pions and muons in the beam do

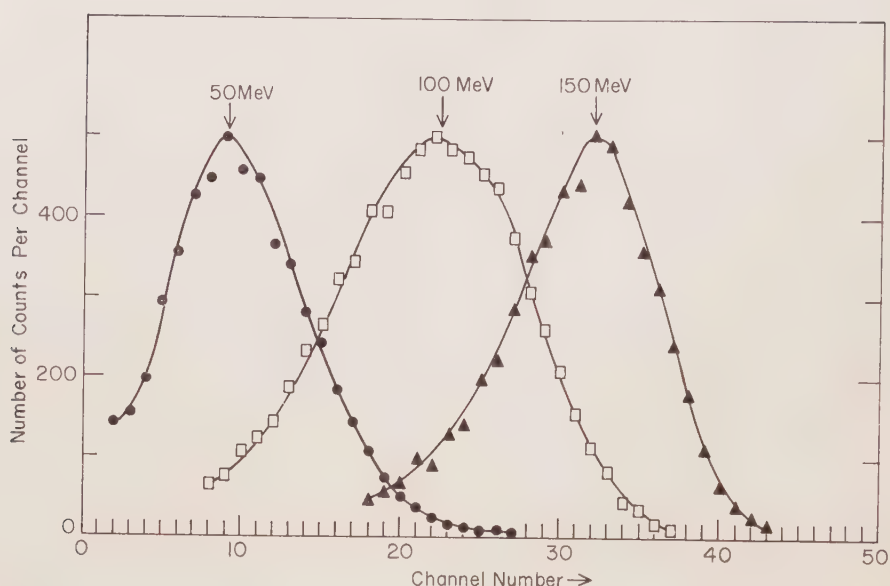


Fig. 4. Pulse-height distributions obtained with 50 MeV, 100 MeV, and 150 MeV electrons. (The solid curves shown simply connect the points and are not the fitted Gaussians.)

not give detectable pulses in the lead glass counter.

Fig. 4 shows three of the pulse-height distributions produced by the electrons from the cyclotron. In fig. 5, the result of fitting Gaussian curves to the observed pulse-height distributions is shown. In the upper part of the figure,  $\sigma$ , the width of the Gaussian in units of channels of the pulse-height analyser, is plotted with its error as a function of the incident electron energy. In the lower part of the figure the centers of the Gaussians together with their errors again plotted as a function of the electron energy. The response of the counter as seen in this last graph is fairly linear and the five lowest energy points have been fitted to a straight line by the method of least squares. The result is

$$C = (0.262 \pm 0.004) E + (-5.9 \pm 0.5) \quad (1)$$

where  $C$  is the number of the channel in the pulse-height analyser and  $E$  is the electron energy in MeV. A non-linearity of the response of the system with electron energy is observed to set in at about 130 MeV. This effect required special treatment in the charge exchange scattering experiments which involve gamma rays of up to 250 MeV in energy<sup>5</sup>). In fitting the

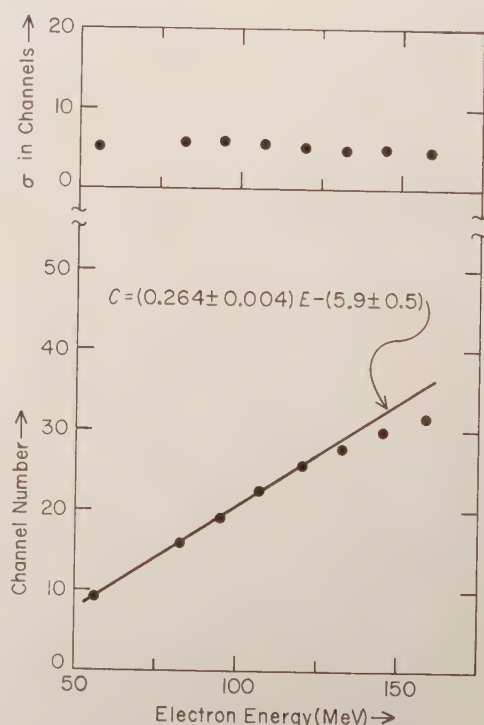


Fig. 5. The upper part of the figure gives the width of the pulse-height distribution,  $\sigma$ , as a function of the energy of the incident electrons. The lower part shows the variation of the centers,  $C$ , of the pulse-height spectra with the electron energy,  $E$ .

<sup>5</sup>) W. Kernan, Ph. D. thesis, University of Chicago (1959).

Gaussian curves to the pulse-height distributions, the data were corrected to account for the non-linearity. It is believed that this non-linearity resulted either from a poor adjustment of the timing of the pulse to be analysed when it arrived at the gate, or perhaps from some limiting action at the gate input due to the size of the pulse. The non-linearity is not considered to be inherent in the operation of the system, but does emphasize the need for a

careful calibration of such a spectrometer before it is applied to a physical problem.

### **Acknowledgments**

It is with great pleasure that we acknowledge the loyal help of the cyclotron staff during the course of this work. Special thanks are due to Mr. W. Kernan, who helped take the data and aided in making the computations, and to Mr. W. Stanula for his loyal assistance.

## A MODIFICATION OF THE NUCLEAR RESONANCE METHOD FOR MEASUREMENTS OF NON-UNIFORM MAGNETIC FIELDS

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It is practicable to obtain accurate measurements of magnetic field intensity in non-uniform fields by the nuclear resonance method with the aid of a set of current-carrying coils which neutralize the field gradient. For use on an electromagnet, a method of controlling the compensating currents by reference to the magnet current is described. This method simplifies the adjustment of the compensating currents. Field gradients up to 150 gauss/cm

may be neutralized by the equipment described, which has been used so far in fields ranging from 2 to 10 kilogauss. In the instruments constructed, it has only been necessary to compensate for variations of the field along the two axes normal to the field itself. However, it should be possible to compensate for variation along the axis parallel to the field as well, without difficulty, and the coil arrangement required for this is described also.

### 1. Introduction

The nuclear magnetic resonance method is a very accurate and satisfactory one for the measurement of magnetic fields, but it has the disadvantage that it cannot be used in non-uniform fields. When different parts of the specimen are in appreciably different fields, they give their resonance output signals at correspondingly different times, as the modulation field varies, and the resulting total signal is reduced in amplitude and spread in time, so that it becomes difficult to observe it and to take accurate measurements on it. The effect of such non-uniformity is less on a smaller specimen, but there is also a loss of sensitivity if the specimen is made too small. The present work was initiated to enable non-uniform fields in certain magnets in this establishment to be measured, in particular, the fields in two double-focussing magnetic particle spectrometers. The method that is used is to compensate the field-gradient locally at the specimen by a suitable set of current-carrying coils, which are arranged so as to give zero field at the centre of the specimen, so that an accurate value of the field at that point may be found. The specimen is also kept

as small as possible, of course, to minimize any second order effects due to non-linearity of the field variation with position. The gradient-compensating coils were added to an ordinary proton resonance magnetic field measuring equipment, of our own design and manufacture, using the marginal oscillator principle†, which was already in regular use within the establishment.

### 2. Description of the Compensating Coils

A total of eight small rectangular compensating coils is used on each probe (see fig. 1). There are two independent sets of four coils, each set being used to compensate a component of the field gradient at right angles to the field itself. The specimen is always placed in the plane of symmetry of the magnets, so that the gradient of the field variation in the direction of the field is zero. The magnitude of the field, a scalar quantity, is the only concern here, when its gradient is referred to. Small changes in the direction of the field do not matter. The arrangement of each set of four coils is shown in fig. 2. This arrangement clearly gives zero field at the centre of the specimen, from considerations of symmetry, while at the same time giving a field gradient proportional to the current. It has

† R. V. Pound and W. D. Knight, *Rev. Sci. Instr.* **21** (1950) 219.

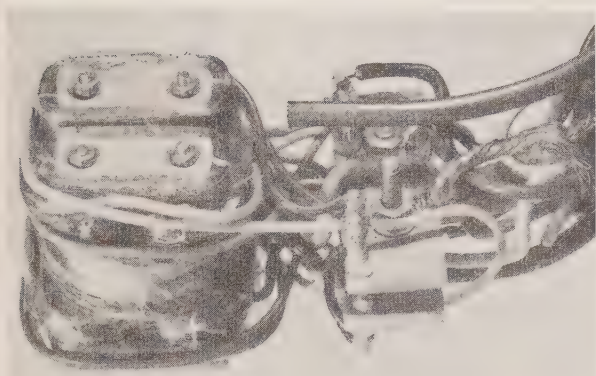


Fig. 1. Magnetometer head complete with two sets of transverse gradient compensating coils.

been found that it is practicable to construct the coils with sufficient accuracy to ensure that the point at which the field is effectively measured is, in fact, closely centred within the specimen. When it is known that the major component of the field gradient will be in a

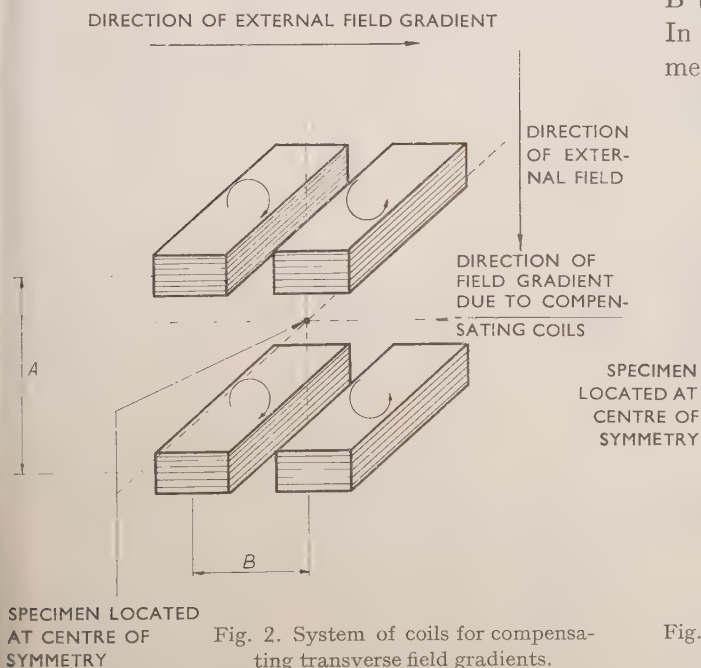


Fig. 2. System of coils for compensating transverse field gradients.

particular direction, for example, in the direction transverse to the beam path in the case of the spectrometers mentioned, it is possible to achieve some economy in size by using fewer turns for the set of coils compensating the other component of the gradient.

In a typical set of coils used, each coil consists of 400 turns of 39 S.W.G. enamel insulated wire, wound onto the edge of a  $\frac{1}{4}$  in.  $\times$   $\frac{3}{4}$  in. rectangular anodized aluminium former,  $\frac{3}{16}$  in. thick. The corners of the former were rounded to  $\frac{1}{16}$  in. radius, to facilitate the winding. The coils were wound on the formers with the aid of a suitable jig, with a layer of paper between the wire and the former, and were secured in place by polystyrene cement. Jets of air from a compressed air supply are directed onto the coils by means of two  $\frac{3}{64}$  in. inside diameter tubes fastened to the stem of the probe (see fig. 1) and with this cooling, such a set of coils can take a current of up to 400 milliamps and compensate a field gradient up to 150 gauss/cm. The equipment has been used so far in fields with intensities ranging from 2 to 10 kilogauss. The dimensions of the specimen used are  $\frac{3}{8}$  in. length by  $\frac{3}{32}$  in. diameter. The spacings A and B (see fig. 2) are  $\frac{21}{32}$  in. and  $\frac{1}{2}$  in. respectively. In its application to the particle spectrometers mentioned, the compensated gradient magneto-

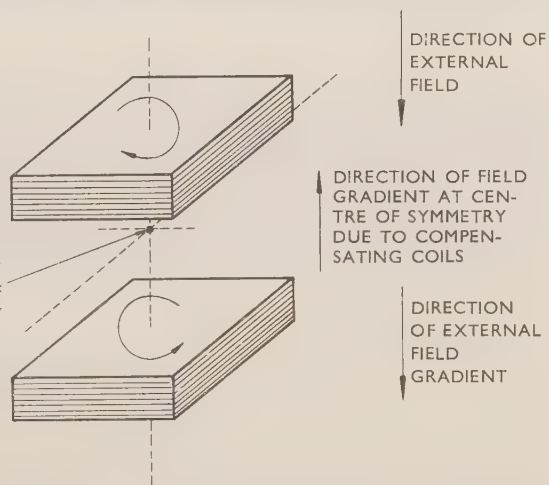


Fig. 3. System of coils for compensating longitudinal field gradients.

meter probe is used inside a vacuum-tight sheath, which may be fixed in a position at the particle detector end where it does not obstruct the beam, or may be inserted into a more central position through a sliding vacuum seal, so that it may be withdrawn when the spectrometer is



in use. The magnetometer has also been used to explore and calibrate the whole field of the magnet in the absence of the vacuum chamber.

Although it has not been necessary in the present application, a gradient of field variation in the direction of the field could be compensated for quite simply, if necessary, by the addition of two extra coils. The arrangement required is shown in fig. 3. Once again, the field at the central point is zero, from considerations of symmetry. Rough calculations based on the replacement of each coil by a magnetic dipole of the same moment indicate that the ampere-turns required to compensate a given field gradient in this case would be comparable with or less than the ampere-turns required for the same gradient transverse to the field, for the same coil areas.

### 3. Control of the Compensating Coil Current

In order to simplify the task of the operator carrying out routine magnetic field measurements with the aid of the compensated-gradient magnetometer, a special method of controlling the current in the compensating coils is provided. This method relates the current in the compensating coils to the current in (or voltage across) the coils of the magnet itself, and thereby to the field and field gradient of the latter. The current in each set of compensating coils may be regarded as consisting of the following three components:

1. A fixed component (subject to preset control).
2. A variable component proportional to the main magnet current (with a proportionality constant subject to preset control).
3. A small component determined by manual fine control.

The circuit by means of which this control is achieved is shown in fig. 4. It is very similar to an ordinary stabilized power supply in its operation, except that its reference voltage is a variable one obtained from the main magnet winding, instead of a fixed one.

It has been found that when the first two adjustments have been properly made for a

given magnet and probe position, very good tracking is obtained, and it has in fact been possible to obtain a good signal over the whole

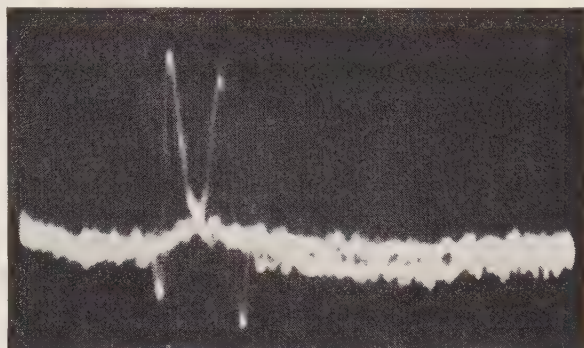


Fig. 5a

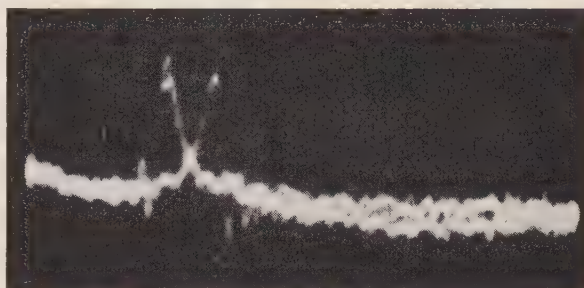


Fig. 5b

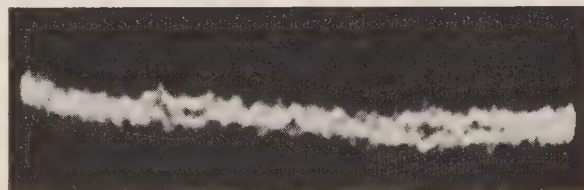


Fig. 5c

Fig. 5. (a) Normal proton resonance signal in a uniform field of 6 kilogauss. (b) Proton resonance signal in a field of 6 kilogauss with a gradient of 70 gauss/cm, after compensation. (c) Proton resonance signal under the same conditions as in 5b, but without compensation.

field range of a probe unit, with little or no adjustment of the manual fine control. (The magnetic field range of each probe is of the order of a field ratio of 1.8 to 1, other probes with coils of more or fewer turns being substituted to extend the range when required.) Once the compensating equipment has been properly set up, therefore, the operator can use the magnetometer as he would in a uniform field, with very

little extra trouble. Any initial difficulty in finding the correct adjustments for the compensating current controls may be minimized by starting with the probe in a part of the field where its gradient is small or predictable, and moving it towards edges or other more difficult points, if necessary, by gradual steps. A typical

example of the results achieved by field-gradient compensation is shown in figs. 5a, b and c. It will be observed that the field-gradient compensation gives a very satisfactory signal in a non-uniform field under conditions such that no signal at all would be visible without it.

## LES CIRCUITS CORRECTEURS DU SYNCHROTRON "SATURNE"

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Reçu le 4 août 1959

The main defects of the magnetic field concern the index value, the position of the magnetic median plane and the residual ripple. For each of the defects, the methods of

correcting are explained as well as their practical realisation and some experimental results are given.

L'équilibre et la stabilité du mouvement des protons exigent que l'induction magnétique  $B$  possède certaines qualités:

– le plan de symétrie des lignes de force, ou plan magnétique médian, doit être confondu avec le plan médian géométrique.

– l'indice du champ,  $n = -(\gamma_0/B_0)(\partial B_z/\partial r)$  doit être compris entre 0.5 et 0.75 (cette condition définit une zone dite zone utile).

– l'induction  $B$  ne doit pas varier azimutale-ment, sauf aux extrémités de quadrants, où elle décroît progressivement, et dans les sections droites, où elle s'annule. La constance de  $B$  pour un rayon  $r$  donné doit être réalisée dans chaque quadrant ainsi que d'un quadrant à l'autre.

– la variation de  $B$  en fonction du temps doit être à peu près linéaire; des écarts importants par rapport à la linéarité pourraient faire perdre des particules au début de l'accélération et gêneraient les utilisateurs du faisceau.

Le champ réel diffère du champ ainsi défini, au moins à certains endroits et à certains instants:

– soit pour des raisons d'ordre mécanique (déplacement ou déformation de certaines parties de l'aimant) soit pour des raisons d'ordre magnétique (influence ou variations de l'induction rémanente et de la saturation), la structure du champ peut se modifier, entraînant des changements du plan magnétique médian ou de l'indice.

– l'induction  $B$  peut varier d'un quadrant à

l'autre; en effet les quadrants, qui sont connectés en série, ne présentent pas la même capacité par rapport à la terre; or la tension d'alimentation de l'aimant a des composantes sinusoïdales. Dès lors la valeur instantanée du courant dans les différents quadrants n'est pas la même. En même temps la loi  $B(t)$  cesse d'être linéaire.

Afin de corriger ces défauts, on a disposé dans l'entrefer des circuits électriques qui doivent créer un complément convenable d'induction au moment et à l'endroit voulus<sup>1</sup>). Ces circuits correcteurs peuvent aussi servir à modifier volontairement la structure du champ magnétique, par exemple à changer brusquement la valeur de l'indice  $n$  dans une partie de la zone utile, ainsi que le nécessite le système d'utilisation du faisceau hors de l'entrefer (éjection du faisceau).

1. Les Circuits Correcteurs de l'Indice  $n$ 

## 1.1. PRINCIPE DE LA CORRECTION

Calculons la variation d'indice  $\delta n$  que produisent en tout point M situé dans le plan médian à l'abscisse radiale  $r$  deux conducteurs  $A_0$  et  $A'_0$  de direction azimutale parcourus par des courants  $I$  de même sens (fig. 1). Nous traitons le problème à deux dimensions car la largeur de la zone utile (0.35 m) est petite vis-à-vis du rayon de l'orbite d'équilibre ( $r_0 = 8.42$  m). Les pièces polaires, supposées parallèles, sont

<sup>1</sup>) J. P. Blewett, M. H. Blewett, W. H. Moore and L. W. Smith, Pole face windings, Rev. Sci. Inst. 24, 9 (1953) p. 773-776

aux cotes  $-h$  et  $+h$ ; leur extension radiale est infinie, ainsi que leur perméabilité. Dans ces conditions, l'induction produite en tout point de l'entrefer par les deux conducteurs est la même que celle que produirait sans la présence du fer, l'ensemble des deux conducteurs et de leurs images dans la surface du fer, tous ces courants étant de même sens et de même intensité<sup>2)</sup>. Cette induction  $b_z$  est parallèle à  $Oz$  comme on le voit en groupant deux par deux

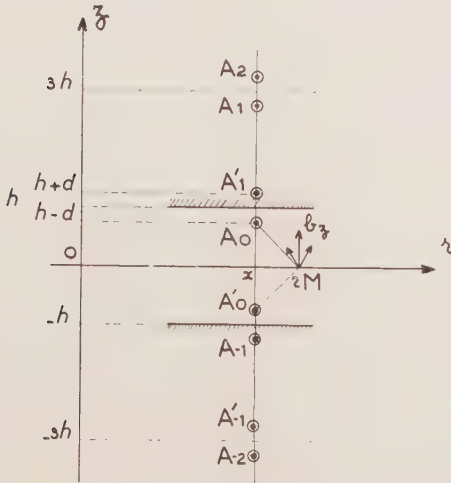


Fig. 1. Schéma de la correction de l'indice  $n$ .

les images symétriques par rapport au plan médian. Le conducteur  $A_0$ , d'abscisse  $x$  et de cote  $h-d$ , parcouru par le courant  $I$ , donne la contribution

$$db_z = \frac{\mu_0 I (r-x)}{2\pi [(h-d)^2 + (r-x)^2]}$$

(nous utilisons la système rationalisé d'unités M K S A). L'ensemble considéré donne, au total,

$$b_z = \sum_{n=-\infty}^{+\infty} \frac{\mu_0 I (r-x)}{2\pi [(h-d+2nh)^2 + (r-x)^2]} + \sum_{n=-\infty}^{+\infty} \frac{\mu_0 I (r-x)}{2\pi [(-h+d+2nh)^2 + (r-x)^2]}$$

la première somme est due aux conducteurs réels ou fictifs  $A_{-n} \dots A_{-1} A_0 A_1 \dots A_n$  et la seconde à  $A'_{-n} \dots A'_{-1} A'_0 A'_1 \dots A'_n$ . En utilisant l'identité

$$\sum_{n=-\infty}^{+\infty} \frac{\lambda}{(\mu+n)^2 + \lambda^2} = \frac{\pi \operatorname{sh} 2\pi\lambda}{\operatorname{ch} 2\pi\lambda - \cos 2\pi\mu}$$

(réf.<sup>3)</sup>) et en remarquant que les conducteurs  $A_0$  et  $A'_0$  sont placés contre les pièces polaires (pratiquement,  $\cos(\pi d/h) = 1$ ), on trouve

$$b_z = \frac{\mu_0 I}{2h} \operatorname{th} \frac{\pi(r-x)}{2h}.$$

On en déduit

$$\delta n = -\frac{r_0}{B_0} \frac{\partial b_z}{\partial r}$$

qui vaut en valeur absolue

$$\delta n = \frac{\pi \mu_0 r_0 I}{4h^2 B_0 \operatorname{ch}^2 \frac{\pi(r-x)}{2h}}.$$

Les deux conducteurs  $A_0$  et  $A'_0$  placés à l'abscisse  $x$  produisent donc dans le plan médian un indice variable en fonction du rayon, maximum au droit des conducteurs ( $r=r_0$ ) et assez rapidement décroissant de part et d'autre. On conçoit qu'il est possible, en utilisant un certain nombre de conducteurs, toujours symétriquement placés par rapport au plan médian, d'adapter suffisamment bien la courbe de la correction d'indice à celle du défaut. La figure 2 donne un exemple d'une telle distribu-

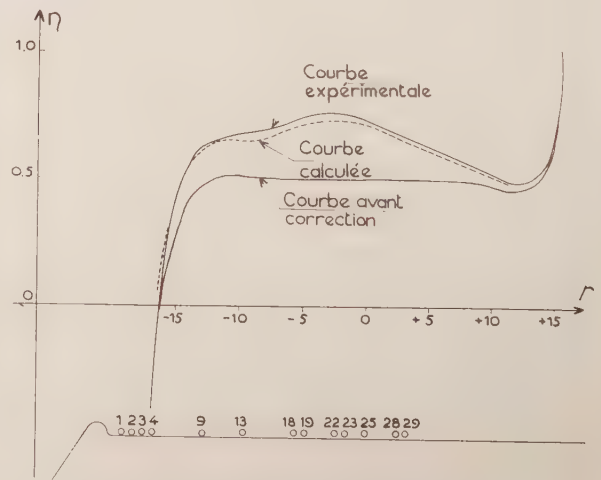


Fig. 2. Un exemple de correction d'indice.

tion; la concordance assez bonne entre valeurs calculées et valeurs mesurées justifie les approximations faites au début du calcul.

<sup>2)</sup> E. Durand, *Electrostatique et magnétostatique* (Masson, Paris, 1953), 541-542.

<sup>3)</sup> E. P. Adams, *Smithsonian mathematical formulas*, (Smithsonian Institution, New-York, 1947) p. 135.

On peut vérifier que la correction est assurée dans un espace assez grand de part et d'autre du plan médian; ce n'est que près des pièces polaires que l'effet de chaque conducteur s'individualise de façon trop marquée (fig. 3).

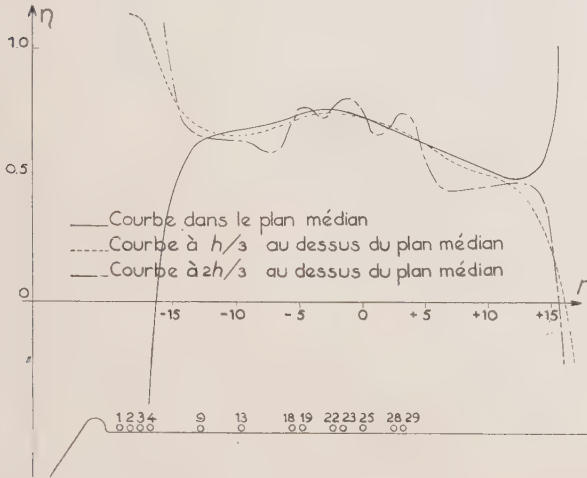


Fig. 3. La correction est perturbée quand on s'approche des pièces polaires.

Dans le cas où l'on utilise une nappe de conducteurs parallèles et équidistants au pas  $p$ , s'étendant sur une largeur radiale  $2l$  de part et d'autre de l'orbite d'équilibre (fig. 4), on obtient l'indice  $\Delta n$  en intégrant  $\delta n$  entre  $r_0 - l$  et  $r_0 + l$  après avoir remplacé  $I$  par  $(I/p) dx$ . On trouve

$$\Delta n = \frac{\mu_0 r_0 I}{2hpB_0} \frac{\text{sh} \frac{\pi l}{2h}}{\left[ \text{sh}^2 \frac{\pi (r - r_0)}{2h} + \text{ch}^2 \frac{\pi l}{2h} \right]}.$$

Cette fonction de  $r$  présente pour  $r_0$  un maximum faiblement accusé qui, dans le cas de Saturne où  $l/h = 2.75$  vaut, à 0.002 près

$$\Delta n_0 = \frac{\mu_0 r_0 I}{hpB_0} \quad (1)$$

$\Delta n$  vaut encore 96% de cette valeur pour  $r = r_0 \pm (l - h)$  c'est-à-dire à un demi entrefer des extrémités de la nappe. On peut donc élever ou abaisser  $n$  d'une façon pratiquement uniforme.

Si l'on veut, par exemple, abaisser  $n$ , il faut créer une induction de correction ayant le

sens des flèches  $d$  (fig. 4) l'induction principale ayant le sens des flèches  $c$ . On voit facilement que les courants correcteurs doivent aller de l'arrière à l'avant de la page. D'où la règle: pour abaisser  $n$ , les courants correcteurs doivent

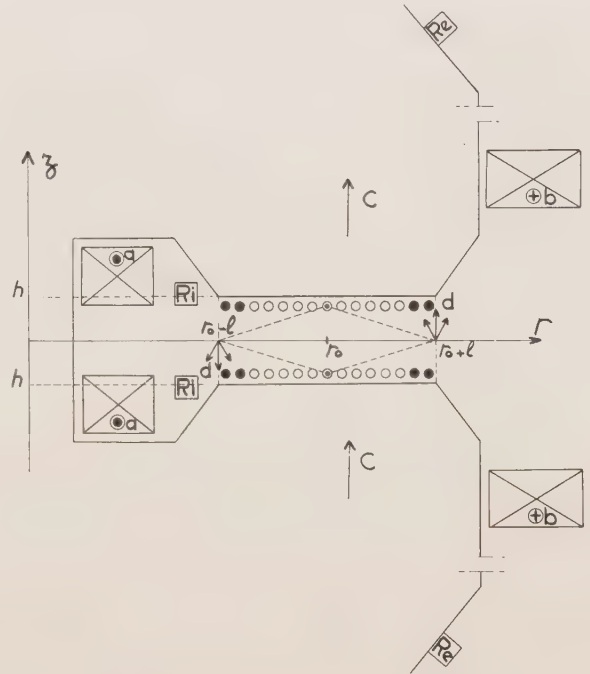


Fig. 4. Position des nappes et des conducteurs de retour ( $R_i$  = retours internes,  $R_e$  = retours externes).

avoir, dans la nappe supérieure comme dans la nappe inférieure, le même sens que le faisceau de protons.

## 1.2. LA CORRECTION D'INDICE AUX CHAMPS FAIBLES

Au début du cycle, la partie rémanente de l'indice fait fortement sentir son influence; l'indice rémanent dépasse 1 dans une grande partie de la zone utile de sorte que l'indice résultant  $n$  atteint ou dépasse encore presque partout 0.7 au moment de l'injection<sup>4</sup>); il semble nécessaire de l'abaisser de  $\Delta n = -0.1$ . Les essais d'injection ont montré qu'il faut l'abaisser davantage ( $\Delta n = -0.18$  à  $-0.22$ ). Cet état de choses, dont la raison n'est pas

<sup>4</sup> G. Bronca, H. Bruck, J. Hamelin, G. Neyret et J. Parain, Etude et contrôle magnétiques du synchrotron Saturne, Onde Electrique (juin 1959).

encore clairement comprise, montre l'intérêt de disposer d'un ensemble assez souple de moyens de corrections.

On pratique donc une correction uniforme au moyen d'une nappe; un courant de 870 mA, calculé par la relation (1) assure un indice de correction valant 0.2 à l'injection ( $B_0 = 32.6 \times 10^{-3}$  Ts)† et décroissant comme  $1/B_0$ .

Ce courant est induit par la variation du champ principal dans un circuit formé des fils de chaque nappe et de conducteurs de retour placés en  $R_1$  et  $R_e$  (fig. 4) à peu près alternativement pour éviter des tensions élevées entre fils voisins. Le déplacement d'un conducteur de retour de  $R_1$  à  $R_e$  change la tension induite de 25 volts par quadrant; un rhéostat en série règle le courant. L'utilisation d'une inductance permet trait de suivre de façon plus précise la loi souhaitable pour  $n(t)$ .

Les fils ont un diamètre de 2 mm; la force sur l'ensemble des 30 conducteurs d'une nappe ne dépasse pas, à 1.5 teslas, 40 newtons (environ 4 kg) par mètre; elle est centripète à la montée du cycle, centrifuge à la descente.

### 1.3. LA CORRECTION D'INDICE AUX CHAMPS ÉLEVÉS

A partir de 1.2 teslas, l'indice croît assez brusquement<sup>4)</sup> à cause de la saturation des pièces polaires. Les écarts par rapport à 0.6 restent à peu près dans le même rapport aux diverses abscisses radiales, ce qui donne la possibilité de corriger à toutes les valeurs de l'induction par une même répartition des conducteurs, à condition de faire croître le courant d'une façon convenable.

La correction  $n_c$  n'est pas uniforme; on a bien constitué une nappe de 60 fils au pas de  $8 \times 10^{-3}$  m mais on n'en utilise que 35, certains autres étant réservés pour différentes corrections. La loi du courant est donnée par le tableau 1; on a  $I = 105 B_0 n_c$ .

La croissance de  $I$  en fonction du temps est imposée par celle de  $B_0$ . Les conducteurs de retour sont placés de façon analogue à ceux du circuit décrit au paragraphe précédent; on a

† On rappelle que 1 gauss =  $10^{-4}$  Ts.

TABLEAU 1

| $B_0$<br>(teslas) | $n_c$ | $I$<br>(ampères) |
|-------------------|-------|------------------|
| 1.15              | 0     | 0                |
| 1.25              | 0.06  | 6.6              |
| 1.35              | 0.16  | 19.9             |
| 1.45              | 0.27  | 41.4             |
| 1.5               |       | environ 60       |

réduit au minimum le couplage par rapport au circuit principal. Les fils ont une section de  $17.8 \text{ mm}^2$ ; la résistance et d'inductance sont, pour un quadrant, 3.1 ohms et 0.13 henrys. Ces éléments déterminent l'alimentation, qui est effectuée au moyen de deux mutateurs hexaphasés de 105 kW; un dispositif électronique règle le courant.

La force sur l'ensemble des 35 conducteurs d'une nappe peut atteindre, à l'induction maximum, 3 200 newtons (environ 320 kg) par mètre; elle est centripète.

## 2. Les Circuits Correcteurs du Plan Médian

Le plan magnétique médian, ou plutôt la surface magnétique médiane, est le lieu des points où la composante radiale de l'induction,  $B_r$ , est nulle. Si le plan magnétique médian est à la cote  $z_0$  par rapport au plan géométrique médian,  $B_r$  est donné au 1<sup>er</sup> ordre par

$$B_r = - \frac{B_0 n (z - z_0)}{r_0} \quad (2)$$

Pour ramener les deux plan en coïncidence il faut créer une induction radiale de correction  $B_c$  égale et opposée à la valeur de  $B_r$  pour  $z = 0$  (fig. 5), soit

$$B_c = - \frac{B_0 n z_0}{r_0}$$

Calculons l'induction radiale que créent en tout point M du plan médian deux conducteurs  $a_0$  et  $a'_0$  parcourus par des courants  $I$  de sens opposés (fig. 6). Un calcul analogue à celui du paragraphe 2.1 montre qu'on obtient en tout point du plan médian, pour les deux conducteurs réels

$$db_r = \frac{\mu_0 I (h - d)}{2\pi [(h - d)^2 + (r - x)^2]}$$

Pour l'ensemble des conducteurs et de leurs images, en tenant compte du sens des courants, en utilisant l'identité

$$\sum_{n=-\infty}^{+\infty} \frac{(-1)^n (\mu + n)}{(\mu + n)^2 + \lambda^2} = \frac{2\pi \sin \pi \mu \operatorname{ch} \pi \lambda}{\operatorname{ch} 2\pi \lambda - \cos 2\pi \mu}$$

et en remarquant que  $\cos \pi d/h$  est pratiquement égal à 1, on trouve

$$b_r = \frac{\mu_0 I}{2h \operatorname{ch} \frac{\pi(r-x)}{2h}}.$$

En utilisant un certain nombre de conducteurs tels que  $a_0$  et  $a'_0$ , on peut ramener à

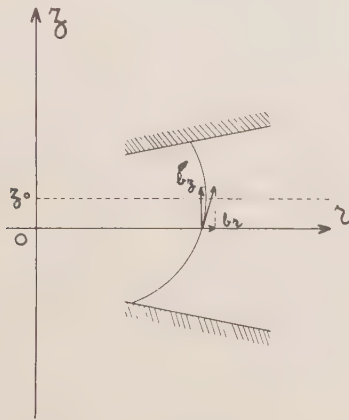


Fig. 5. Plan médian géométrique ( $z = 0$ ) et plan médian magnétique ( $z = z_0$ ).

chaque abscisse radiale le plan magnétique médian en coïncidence avec le plan géométrique médian.

Dans le cas particulier d'une translation verticale  $z_0$ , on prend une nappe de fils équidistants au pas  $p$ , s'étendant de  $r_0 - l$  à  $r_0 + l$ ; on obtient l'induction correctrice  $B_c$  en intégrant  $b_r$  après avoir remplacé  $I$  par  $(I/p) dx$ :

$$B_0 = \frac{\mu_0 I}{\pi p} \left[ \text{Arc tg sh } \frac{\pi (r - r_0 + l)}{2h} - \text{Arc tg sh } \frac{\pi (r - r_0 - l)}{2h} \right].$$

Cette fonction de  $r$  présente pour  $r_0$  un maximum faiblement accusé qui, dans le cas de Saturne vaut à 1.8 pour cent près  $\mu_0 I / p$ ;  $B_e$  vaut encore 87 pour cent de cette valeur pour  $r = r_0 \pm (l - h)$  c'est-à-dire à un demi-

entrefier des extrémités de la nappe. On peut donc élever ou abaisser le plan médian, d'une façon presque uniforme d'une quantité  $\Delta z_0$  donnée, en valeur absolue par

$$\Delta z_0 = \frac{\mu_0 r_0 I}{n p B_0}. \quad (3)$$

Il est facile de voir que pour abaisser le plan médian, les courants correcteurs doivent avoir, dans la nappe inférieure, le même sens que le faisceau de protons, et le sens inverse dans la nappe supérieure.

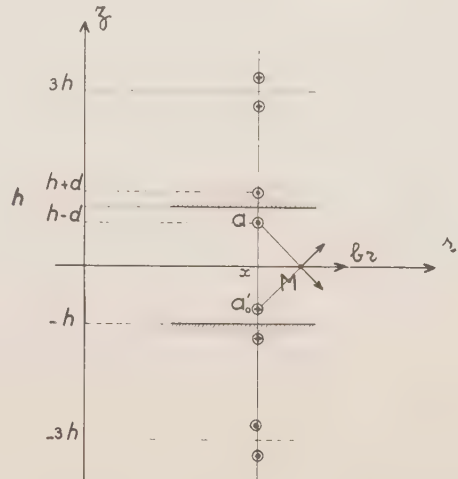


Fig. 6. Schéma de la correction du plan médian magnétique.

### 3. La Correction des Ondulations

L'alimentation de l'aimant est constituée de deux groupes hexaphasés produisant un courant dodécaphasé. Après redressement il subsiste des composantes alternatives. La fréquence de l'alternateur étant de 70 Hz, les ondulations de chacun des groupes sont à 420 Hz. Les bobines de l'aimant ayant une capacité par rapport à la masse et le point milieu des groupe étant à la masse, le taux d'ondulation n'est pas le même dans les différents points du circuit, et en particulier entre les quadrants<sup>5)</sup>.

Il est possible de corriger les ondulations seulement dans la zone utile de l'aimant<sup>6)</sup> en

<sup>5)</sup> G. Bronca, H. Bruck, G. Gendreau et M. Salvat, Etude de l'injection dans Saturne, Onde électrique (juin 1959).

<sup>6</sup>) H. G. Heard, A new method for controlling the magnetic field in the aperture of synchrotrons (University of California, 1956) UCRL 3427.

utilisant certains fils situés aux extrémités des nappes des circuits correcteurs. Ce circuit de correction est mis en série avec une capacité dont le rôle en première approximation est de

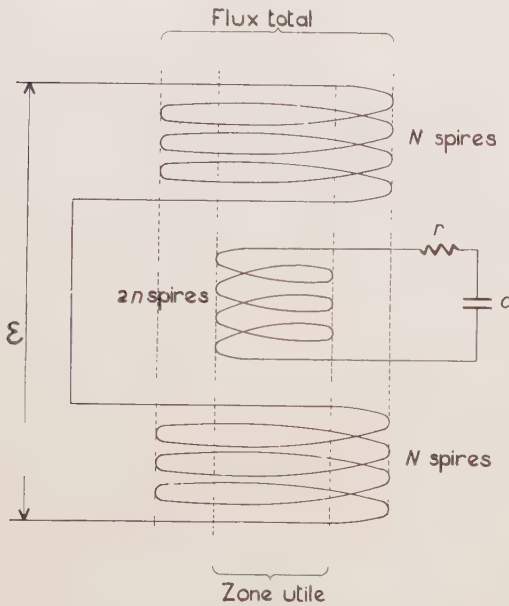


Fig. 7. Schéma de l'atténuation des ondulations.

bloquer la composante continue de la tension (fig. 7).

On définit un coefficient d'atténuation de l'ondulation

$$\frac{B}{B'} = \frac{NI + ni}{NI'}$$

dans lequel  $B$  et  $B'$  désignent l'induction après et avant correction,  $2N$  le nombre de spires du circuit d'excitation parcouru par le courant  $I$  lorsque le courant de correction est  $i$  dans le circuit constitué de  $2n$  spires.  $I'$  est le courant d'excitation en l'absence de correction.  $I$ ,  $I'$ ,  $i$  sont les courants sinusoïdaux produits par l'ondulation de tension, de pulsation  $\omega$  et d'amplitude  $\varepsilon$ ,  $Z$  et  $z$  sont les impédances des circuits d'excitation et de correction,  $M$  le coefficient d'induction mutuelle entre les deux circuits.

$$i = -\frac{jM\omega\varepsilon}{Zz + M^2\omega^2},$$

$$I = \frac{z\varepsilon}{Zz + M^2\omega^2}, \quad I' = \frac{\varepsilon}{Z}$$

le facteur d'atténuation de l'ondulation a pour expression

$$\frac{B}{B'} = \frac{Z}{N} \frac{(zN - jM\omega n)}{(Zz + M^2\omega^2)}$$

en posant  $Z = jL\omega$  et  $z = r + jl\omega - j/c\omega$ .

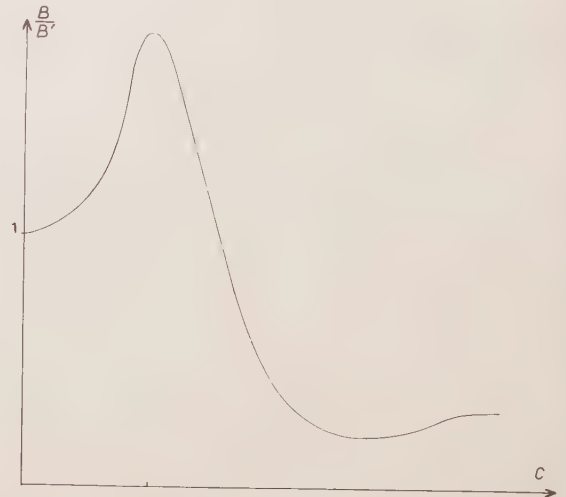


Fig. 8. Taux d'ondulation après correction, en fonction de la capacité.

Prenons le carré des modules

$$\left(\frac{B}{B'}\right)^2 = \frac{\left[\omega\left(M\frac{n}{N} - l\right) + \frac{1}{c\omega}\right]^2 + r^2}{\left[\omega\left(\frac{M^2}{L} - l\right) + \frac{1}{c\omega}\right]^2 + r^2}.$$

L'allure de la courbe est la suivante (fig. 8) en fonction de la capacité en remarquant que

$$\frac{M}{L} < \frac{n}{N},$$

c'est à dire que le circuit correcteur n'embrasse qu'une partie du flux principal

En faisant l'hypothèse que tout le flux passant dans la spire de correction passe dans le courant principal, on a

$$l = \frac{n^2}{N^2} kL \quad \text{et} \quad M = \frac{n}{N} kL$$

$k$  étant un coefficient qui dépend de la disposition des conducteurs; alors  $l/M = n/N$ .

En fait, on aura  $l > M(n/N)$  à cause du flux se refermant autour de la boucle de correction. En appelant  $\rho$  la résistance d'une spire double, on a  $n\rho = r$ .

$(B/B')^2$  calculé maintenant dans le cas où  $C$  est infiniment grand, vaut

$$\left(\frac{B}{B'}\right)^2 = \frac{(np)^2}{\left[\omega \frac{n^2}{N^2} Lk (k-1)\right]^2 + (np)^2} \quad (4)$$

Pour un quadrant, les valeurs sont  $p = 0.06\Omega$ ,  $n = 2$ ,  $N = 32$ ,  $L = 0.3H$ ,  $k = 0.7$ ,  $\omega = 2.6 \times 10^3$ , et l'on trouve  $B/B' = 0.06$ .

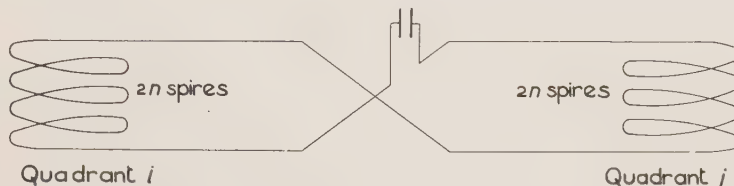


Fig. 9. Schéma de l'atténuation du déséquilibre entre deux quadrants.

Un procédé analogue peut être utilisé pour égaliser les différences d'induction entre les quadrants. Les circuits de correction sont montés en opposition. Dans ce montage, les capacités pourraient être supprimées si les boucles de correction étaient rigoureusement de même surface (fig. 9). Un montage permet d'étendre l'égalisation aux 4 quadrants de l'aimant (fig. 10). On définit un coefficient d'égalisation  $(B_i - B_j)/(B'_i - B'_j)$  entre les quadrants  $i$  et  $j$ ;  $B'_i$  et  $B'_j$  étant les inductions avant correction. Les calculs précédents sont directement applicables à ce montage.

Un essai a été effectué en utilisant deux spires doubles des nappes de circuits correcteurs

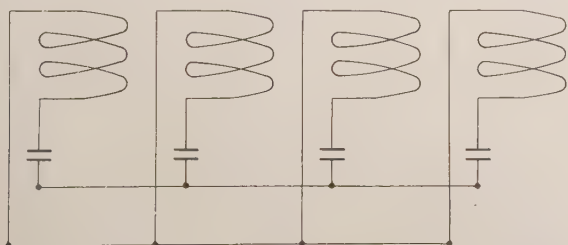


Fig. 10. Schéma de l'atténuation du déséquilibre entre les quatre quadrants.

gros fils (ce sont les 8 fils dessinés en noir sur la fig. 4). Chacune des capacités valaient  $900 \mu F$ .

Dans cet essai on a mesuré la différence des inductions entre deux quadrants en intégrant

la différence des tensions de deux bobines placées au centre de l'entrefer de chacun des quadrants à étudier. L'enregistrement est effectué par un oscillographe.

La figure 11 donne  $B_1 - B_2$  en fonction du temps pour les instants voisins de l'injection. Le coefficient d'égalisation est voisin de 0.2, le calcul prévoyait un coefficient d'égalisation de 0.06 pour une capacité infinie. Cette différence

provient de l'hypothèse du passage de tout le flux dans la spire de correction ainsi que de l'imprécision qui affecte les valeurs numériques utilisées dans l'application de la formule (4).

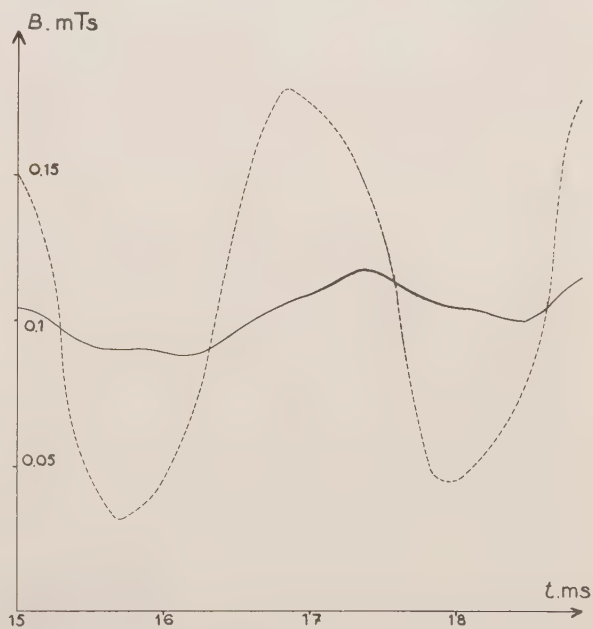


Fig. 11. Un exemple d'atténuation du déséquilibre entre deux quadrants.

## Remerciements

Nous exprimons nos remerciements à MM. Lecroart et Mérard qui ont contribué à la correction des ondulations, et à M. Fournier qui a effectué une partie des mesures.

## THE USE OF ELECTROSTATIC GENERATORS AS INJECTORS FOR ELECTRON SYNCHROTRONS<sup>†, ††</sup>

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Some experiences with the use of electrostatic generators as injectors for electron synchrotrons at Cornell University and M.I.T. are described. At M.I.T. the injection energy was changed without any other essential modifications to

the synchrotron. A factor of ten in intensity has been obtained, along with greater stability. Space charge does not appear to be the limiting mechanism of the beam current.

One of the uses for a low energy accelerator is to provide a source of particles for injection into a higher energy machine. The following paper will describe some experiences using electrostatic generators as injectors for electron synchrotrons.

After World War II many electron synchrotrons and betatrons were built of which the largest had peak energies of 300 MeV. That energy was chosen to be above the threshold for  $\mu$ -meson pair production; the  $\pi$ -meson was unknown at the time. The 300 MeV machines had approximately the same parameters: the radii were about 1 meter and the apertures of the order of 10 cm wide by 5 cm high<sup>1</sup>). Electrons of 100 kilovolt energy (350 keV/c momentum) were injected into magnetic fields of about 10 gauss. The electron guns were very simple, consisting of vertical filaments behind a vertical slot. In general, they sprayed peak currents of several amperes into a large solid angle. Only a small fraction of the injected current was accepted for acceleration. Operation of these machines was often erratic. The 300 MeV machine at MIT accelerated at best about  $4 \times 10^8$  electrons/pulse and more usually about  $10^8$  electrons.

Most of the injected electrons strike the injector after several turns and many schemes with time-dependent perturbations were introduced to facilitate the capture of electrons into stable orbits<sup>2</sup>). In general, it was found that these perturbations made it easier to obtain a sizeable beam, but that the fractional increase of intensity from the perturbation became smaller as other parameters became better adjusted (such as the shape of the magnetic field). This saturation of output with injected beam was blamed on such things as poorly shaped magnetic fields, improper gun design and space-charge blow-up.

The peak energy of 300 MeV turned out to be an unfortunate choice since the peak of the ( $\frac{3}{2}, \frac{3}{2}$ ) resonance of photo-meson production is near that energy. Hence physicists urgently wanted higher energy machines and because of the small cross-sections of photo-meson production wanted higher intensity machines. A higher energy machine requires a larger radius because saturation effects limit the maximum field to about 12 kilogauss. An injection field of about 10 gauss is required to minimize the effects of remnant fields. Hence the injection momentum must be of the order of one-

<sup>†</sup> This work is supported in part through AEC Contract AT(30-1)-2098 by funds provided by the U.S. Atomic Energy Commission, the Office of Naval Research and the Air Force Office of Scientific Research.

<sup>††</sup> Based on an invited paper presented at the High Voltage Engineering Corporation Accelerator Conference, October, 1958.

<sup>1</sup>) J. E. Thomas, Jr., W. L. Kraushaar and I. Hapern, — High Energy Accelerators—Synchrotrons, Annual Reviews of Nuclear Science, Vol. 1 (1952) p. 175.

<sup>2</sup>) J. Moffatt, J. Sci. Instr. 35 (1958) 447; This paper contains a list of references to other time dependent injection schemes.

thousandth of the final momentum. In addition, if one wants a higher energy machine without the cubic increase with radius of the cost of the magnet one must reduce the relative aperture and keep higher tolerances.

The success of the Cosmotron at Brookhaven, in particular measurements of the required apertures, inspired Prof. R. R. Wilson at Cornell to design a 1.2 GeV synchrotron using the existing power supply from the old 300 MeV machine. Because of the Cosmotron's success with a Van de Graaff generator, and their commercial availability, a 2 MeV Van de Graaff was chosen for an injector†. The synchrotron has a radius of  $12\frac{1}{2}$  ft, the vacuum chamber or donut is of elliptical cross-section ( $3\frac{1}{2}$ " by  $1\frac{3}{8}$ ") with about  $\frac{1}{4}$ " walls. The magnet is of the strong focusing type with approximately 3 betatron oscillations per turn. The circulation time is about 100 nanoseconds so that a single turn could be injected and successive turns followed. The vacuum chamber is constructed of glass and the first turn could be seen in detail by means of a movable fluorescent screen. The first 20 turns after injection could be studied by using electrons scattered from a tungsten cobweb located in one of the straight sections. Latter turns could be detected by means of an induction pickup. Hence the intensity of the beam could be monitored during the entire acceleration cycle. There is a rapid decrease on the first several turns, because of the small aperture, followed by a slow decrease until the electrons have about 10 MeV energy, after which the losses are very small.

The Cornell machine operating at 1 GeV and 30 pulses per second delivers about  $5 \times 10^8$  electrons/pulse, (estimated from the induction probe and the gamma-ray intensity). They have found that only about  $1\frac{1}{2}$  turns can be accepted and that saturation of output intensity with input occurs at currents of about 30 mA of injected beam. Currents up to several hundred milliamperes have

been tried. Several different injections schemes have been tried with the same final results<sup>3</sup>).

At MIT, two lines of improvement were followed: (1) building a new high-energy machine for 6 GeV (jointly with Harvard) and (2) increasing the intensity of the existing 300 MeV machine. To increase the intensity of the 300 MeV machine, a 1 MeV Van de Graaff generator was installed in place of the 100 kV pulse transformer††. A 1 MeV electron has about four times the momentum of a 100 keV electron, so that the injection field is about 40 gauss. Hence the effects of remnant fields should be smaller. Van de Graaff beams can be well focused and hence it was hoped to direct a larger fraction of the injected charge into the right orbit. Moreover, at high energy the space-charge effects should be smaller. An estimate of the space-charge effects can be obtained by considering the beam to be a uniformly charged cylinder. The repulsion of like charges will tend to expand the beam and the pinch effect (attraction of parallel currents) will tend to contract the beam. The electromagnetic force acts in the same manner as the focusing component of the guide field and should cause changes in the gradients which would increase the amplitude of oscillation and if large enough should result in instability. For a given machine, the gain in charge should go as  $\gamma^3\beta^2$  if the beam has the same geometry. For the MIT 300 MeV machine, the space-charge limit increases by a factor of 50 in going from 100 keV to 1 MeV injection.

The Van de Graaff accelerator is located on a tangent to the donut. The beam is brought to the synchrotron through a series of deflecting, translating, and focusing coils. The beam is led into the magnetic guide field by an electrostatic inflector; the inflector consists of two parallel plates that produce an electric field which balances the magnetic field. An orbit contracting pulse is applied to a turn about the flux bars during injection<sup>4</sup>). Initial acceleration is still by betatron action.

A single turn pulse was injected in order to

† Manufactured by the High Voltage Engineering Corporation, Burlington, Mass.

†† Manufactured by the High Voltage Engineering Corporation, Burlington, Mass.

<sup>3</sup>) Private communication by Prof. Raphael Litteaur.

<sup>4</sup>) J. Moffatt, J. Sci. Instr. 35 (1958) 447.

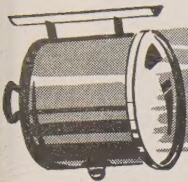
follow the initial turns. A pulse of about 15 nanoseconds was generated by discharging a short length of coaxial cable into its characteristic impedance. Measurements of the acceptance times keeping the magnetic field corrections constant showed that the Van de Graaff energy could vary by  $\pm 2$  keV, corresponding to contributions from about 6 turns. The Van de Graaff machine is regulated to better than  $\pm 500$  V by feedback to the bottom inducting plate. If, however, the Van de Graaff voltage and injection time are changed together, one finds that one can accept over  $\pm 12$  keV without changing any other controls, including the orbit contractor timing. On the basis of this measurement, an insulated lining was installed inside the Van de Graaff pressure tank and the capacity coupling between this liner and the high voltage terminal was used to induce a 20 kilovolt step on the top terminal of the Van de Graaff. From the single turn measurements one would expect an intensity gain of about six. In practice the liner pulse has acted like other time dependent perturbations, with a saturation of output occurring at about the same level as without it. For small injected currents a large increase of intensity is possible, larger injection currents give less fractional increase. The best beam to date has been obtained with this liner and is about twice the best beam without the liner. As larger beams are obtained less magnetic

contraction is needed. At present an oxide-coated button mounted in a Pierce type gun is used<sup>5)</sup>. The voltages of the top two electrodes of the Van de Graaff can be varied for focusing. One-half microsecond pulses of 200–1000 mA can be obtained. The beam at the base of the Van de Graaff has a diameter of about  $\frac{1}{2}$ " at 200 mA. As one goes to higher currents one can see space-charge blow-up. It has been observed that saturation of accelerated beam is 10 times the old maximum or about  $4 \times 10^9$  electrons/pulse†. There has been a great improvement in stability and reliability and a decrease in the sensitivity to the magnetic-field corrections. On the other hand, the intensity increase has not been that expected from the space-charge limit. Apparently space-charge is not the limiting phenomena. Perhaps there is some other limiting instability in non-cylindrical electron beams. At present this limit is still a puzzle. One might hope that synchrotron capture efficiencies might be increased further by understanding the limiting mechanism.

As for the future, where high energy and high intensity are desired the synchrotron will ultimately give place to the linear accelerator. A linear accelerator might give up to  $10^{14}$  electrons/pulse. At high energy, the radiation loss of a circular machine puts an upper limit to the size of a circular electron machine of about 10 GeV. At lower energies, synchrotrons cost less for a given energy and deliver a spread out beam instead of a micro-second pulse with the corresponding improved duty cycle.

† This figure is estimated from the maximum observed gamma ray intensity of  $8 \times 10^8$  equivalent quanta per pulse from a target with an effective thickness of 0.2 radiation lengths. Only about one-eighth of the intensity is in a useful cone. (Within 5 milliradians.) Average useful intensities are about one half of the Maximum, i.e.,  $5 \times 10^7$  equivalent quanta per pulse.

<sup>5)</sup> G. Davidson, S. Ozaki and R. Weinstein, The use of Oxide Coated Cathodes in High Current Electron Accelerators. (To be published.)



# Spotlight on Industry

This section appears each month and features various happenings of interest in the nuclear industrial world and objective information on products of the nuclear manufacturers. It is offered entirely as a service and all nuclear

manufacturers are cordially invited to submit clearly objective and descriptive material on their endeavours and product developments for consideration for use in this section.

## KRYPTON NUCLEAR BATTERIES

*Radiation Research Corporation 1114  
First Avenue, New York 21, N.Y.*

Radiation Research Corporation announces the availability of krypton batteries producing output potentials greater than 5 kV in a volume less than one-quarter of one cubic inch and free from radiological ingestion hazard. The first commercial nuclear batteries (ref. <sup>1</sup>) utilizing strontium 90 as a source of beta particles, have operated for over five years and, as the half-life of strontium 90 is 25 years, much longer life is expected.

Unfortunately, however, strontium 90 is toxic to human beings and the quantity of isotope used in each battery is over 1000 times the maximum amount permissible in the body. To overcome the toxicity problem, the radioisotope krypton 85 has been substituted for strontium 90. Krypton is not metabolized in any way by the body; therefore, batteries using krypton are hazardous only if unshielded. Furthermore, if a krypton battery is accidentally opened the krypton dissipates harmlessly into the atmosphere.

The operation of krypton and strontium batteries is based upon the principle that beta particles penetrating a solid dielectric are collected as electrical energy. The dielectric used in krypton batteries is the same radiation-improved dielectric previously used in long-lived strontium batteries (see ref. <sup>1</sup>).

<sup>1</sup>) John H. Coleman, Radioisotopic High-Potential, Low-Current Sources, *Nucleonics* (December 1953) pp. 42-45.

## 4" PRECISION ELECTROMAGNET FOR LABORATORY APPLICATIONS

*Applied Radiation Corporation, 2404  
North Main Street, Walnut Creek,  
California*

A line of precisely constructed electromagnets has been developed for use in educational and research institutions where accurately controlled magnetic fields are required.



### Specifications

|                                 |                                                                |
|---------------------------------|----------------------------------------------------------------|
| a. Coil current                 | 2.5 amps (max)                                                 |
| b. Coil voltage                 | 300 volts (hot)                                                |
| c. Coil insulated to ground for | 600 volts (min)                                                |
| d. Total number of turns        | 11 000                                                         |
| e. Coil resistance              | 132 ohms (hot)                                                 |
| f. Cooling water requirements   | 0.7 gpm ( $\frac{3}{8}$ " OD Copper Tube Connectors)           |
|                                 | 25 psig (Min), 60 psig (max) (clean tap water is satisfactory) |

|                                           |                                                                                                                                                                                                                                        |
|-------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| g. Gap width range (adjustable gap model) | LM4-AG, 4" dia. pole cap 0-4 $\frac{1}{2}$ "<br>LM4-AG, 6" dia. pole cap 0-2 $\frac{1}{2}$ "<br>LM4-AG, 8" dia. pole cap 0-2"                                                                                                          |
| h. Gap width range (fixed gap model)      | LM4-FG, 4" pole cap 0-3" (gap width to be specified by customer)<br>LM4-FG, 6" pole cap 0-2 $\frac{1}{2}$ " (gap width to be specified by customer)<br>LM4-FG, 8" pole cap 0-2 $\frac{1}{2}$ " (gap width to be specified by customer) |

### i. Weight:

|                        |          |
|------------------------|----------|
| 1. LM4-AG, 4" pole cap | 595 lbs. |
| 2. LM4-FG, 4" pole tip | 555 lbs. |

## NEW SUNVIC RELAY FOR HIGH SPEED VALVE OPERATION

*Sunvic Controls Ltd., Harlow, Essex*

A booster relay, Model 61H, has been introduced which, it is stated, will improve design of process control loops by making it possible to increase valve stroking speeds considerably without loss of stability.

Major time lags in any automatic control loop occur at the diaphragm control valve, where considerable volume flows of air have to be handled. For example a 1" stroke valve, with 64 sq.ins. diaphragm top, might take half a minute to stroke fully when receiving its signal directly from a controller. This time can be reduced by using a valve positioner incorporating volume amplification but time con-

stants will still average approximately 10 secs. Also volume amplification can lead to instability and oscillation.

Sunvic state, however, that using the new 61H relay in conjunction with their high precision valve positioner applied to a 1" stroke valve with 64 sq.ins. diaphragm top, it is possible to stroke the valve fully in 1.5 sec without any tendency to oscillation. A high degree of stability is achieved because the relay incorporates an adjustable damping feedback circuit which permits tuning response to any particular circuit.

The relay will give a 10.3 s.c.f.m. average flow for 3-15 psi change and 13.5 s.c.f.m. for a 15-3 psi change. When used with Sunvic's own valve positioner, 0.7% of full valve stroke causes 12 psi valve pressure change. The relay can be used without a valve positioner when speed, but not positioning accuracy, is essential.

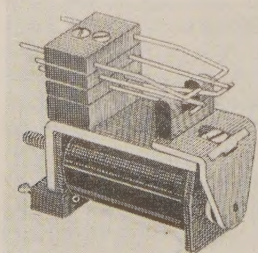
The model 61H will accept input pressure up to 100 psi so that far higher valve stroking speeds than those quoted above are possible. It will work in ambient temperatures from  $-40^{\circ}\text{F}$  to  $+180^{\circ}\text{F}$ . Dead spot is within 0.05% of full scale; linearity within 0.4% of full scale and reproducibility, when used with Sunvic valve positioner and with 4% valve friction, is within 0.1% of full scale.

Maximum air consumption under balanced conditions is 0.5 s.c.f.m.

#### MINIATURE LOW CAPACITANCE G.P. RELAY

*Ericsson Telephones Ltd., Beeston, Nottingham*

This relay is intended for use in circuits requiring a small two change-over relay with low capacitance between springs, and between springs and frame. The conventional flat springs have been replaced by wire springs mounted in moulded blocks.



Typical performance characteristics at 1000 c/s give a capacitance of not worse than  $1\ \mu\text{F}$ d between all springs and not worse than  $2\ \mu\text{F}$ d between springs and frame.

#### Dimensions

Overall Dimensions (including tags).

$1.54 \times 1.09 \times 0.48$  ins.

$(38.9 \times 27.8 \times 12.3)$  mm.

When fitted with a perspex protecting cover for the springset.

$1.56 \times 1.31 \times 0.63$  ins.

$(39.6 \times 33.3 \times 15.9)$  mm.

#### Weight

Relay 30 g

Cover 6 g

#### Operate Conditions

A typical design for a low Capacitance G.P. Relay to operate on a nominal voltage of 24 volts is compared with a normal G.P. Relay as follows:

#### Low Capacitance G.P.

Nominal Coil Resistance . . . . . 700 ohms

Nominal Turns On Coil . . . . . 8000

Minimum Circuit Operate Current 22.2 mA

#### Normal Type G.P.

Nominal Coil Resistance . . . . . 1000 ohms

Nominal Turns On Coil . . . . . 9700

Minimum Circuit Operate Current 15.6 mA

#### Temperature Category

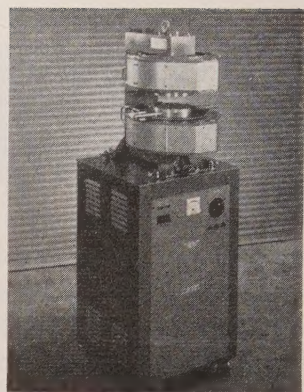
Suitable for operation in Ambient Temperatures in the  $-40^{\circ}\text{C}$  to  $+85^{\circ}\text{C}$  range.

#### Coil Ratings

The maximum power dissipation for the fully wound relay in an ambient temperature of  $15.6^{\circ}\text{C}$  is 1.5 watt. At higher ambient temperatures the wattage must be appropriately lowered.

#### Insulation

The dielectric breakdown test voltage between all parts not in electrical contact is 1000 volts R.M.S. at 50 c.p.s.



#### Specifications

|                                                             |                                                                                                                 |
|-------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| Current range                                               | 0-2.5 amps (four overlapping ranges)                                                                            |
| Voltage range                                               | 0-300 volts                                                                                                     |
| Ripple                                                      | 0.2% (max. peak to peak)                                                                                        |
| Regulation for 10% change in load impedance or line voltage | 0.2% for current output between 2.5 amps and 0.2 amp.                                                           |
| Size:                                                       |                                                                                                                 |
| Power Supply                                                | 19" wide<br>15" deep<br>28" high                                                                                |
| Control Panels                                              | 19" wide<br>17" deep<br>19 1/4" high                                                                            |
| Weight                                                      | 250 lbs.                                                                                                        |
| Input Power                                                 | 208 volts, 3 phase, 4 wire (grounded neutral) 60 cps, 1.5 kVA. One phase will also drawn an additional 0.5 kVA. |

Air ventilation is recommended if power supply and regulator are mounted in a closed cabinet.

#### CURRENT-REGULATED POWER SUPPLY

*Applied Radiation Corporation, 2404 North Main Street, Walnut Creek, California*

An electronically controlled D. C. power supply has been developed for use with laboratory magnets such as ARCO Model LM4 where low ripple and constant current are required.

#### ULTRA-SENSITIVE MASS SPECTROMETER LEAK DETECTOR

*Smith & Dorian inc., 39 West 55th Street, New York 19*

A new ultra-sensitive mass spectrometer leak detector, with important applications in vacuum technology, has been introduced by the Crosby-Teletronics Corporation.